DIRECT VERSUS INDIRECT METHODS FOR THE ESTIMATION OF VARIANCE-COVARIANCE MATRICES AND REGRESSION PARAMETERS WHEN DATA ARE SKEWED AND INCOMPLETE

By

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In memory of my beloved father, Jamu Mayotcha Chirembo,
mother, Sellah NyaMalamba NyaGondwe, and
brother, Ronald Mayotcha Chirembo. Rest in Peace!

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DIRECT VERSUS INDIRECT METHODS FOR THE ESTIMATION OF VARIANCE-COVARIANCE MATRICES AND REGRESSION PARAMETERS WHEN DATA ARE SKEWED AND INCOMPLETE

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Five estimation methods, complete data only (CD), mean replacement (MR), complete pair variables only (PP), regression (RG), and expectation-maximization (EM), were compared with respect to their accuracy in the estimation of missing values, variance-covariance matrices, and regression coefficients. The effect on estimation accuracy was investigated under amounts of missing data from both independent and dependent variables, degree of data skewness, type of missing data, average correlation of variables, residual variance, number of independent variables, and sample size.

Overall, the EM is superior to the rest of the estimation methods. However, the MR method is a very good alternative, and in most situations it performs just as well as the EM. The CD method performs as well as the EM method when regression coefficients are estimated. The performance of all methods declines when data are censored and the percentage of missing data is high (15% missing data). The PP method performs as well as the EM and MR methods when the independent variables are only slightly correlated (average correlation of 0.2). The RG method performs well when the number of independent variables is low (three independent variables); otherwise it is the worst compared to the rest. Estimation is better when data are missing at random than when data are censored.

CHAPTER 1
INTRODUCTION

In the absence of missing values, many multivariate statistical analyses, including least squares regression, factor analysis, and discriminant analysis, are based on an initial reduction of the data to the sample mean vector, $\hat{\mu}$, and sample variance-covariance (V-C) matrix, $\hat{\Sigma}$, of the variables. When vector-valued observations are made, it is not unusual for one or more of the elements in one or more of the vector-valued observations to be missing. The question of how to estimate Σ, μ, and other population parameters when there are missing data is, therefore, an important one. If the number of missing entries in the data matrix is very small, it may be reasonable to discard all the measurements for a given individual with missing entries and proceed with the analysis as if that data had never been collected. However, even if the percentage of missing observations is small, the amount of data excluded by this procedure can become substantial if the number of variables is large.

There are three classes of methods for handling incomplete data, namely the complete data (CD) method (Timm, 1970), the indirect methods, and the direct methods. In the

CD method the complete observations are used in standard procedures such as maximum likelihood (ML) or least squares (LS) to estimate parameters. In the indirect methods the missing entries are first estimated and then the estimated and observed data are used in standard procedures to estimate the parameters. Among the indirect methods are the mean replacement (MR) method (Wilks, 1932), the complete pair only (PP) method (Glasser, 1964), the principal component (PC) method (Gleason & Staelin, 1973), and the regression replacement (RG) method (Buck, 1960). In the direct methods both the complete and incomplete observations are used to estimate the parameters by a maximum likelihood procedure.

Most early work on direct estimation focused on certain special patterns of missing observations; for example Anderson (1957), Afifi and Elashoff (1966), and Hocking and Smith (1968). Subsequent work by Orchard and Woodbury (1972) did not rely on special patterns. Orchard and Woodbury used a special case of the Expectation-Maximization (EM) algorithm. A variant on the direct method, called pairwise maximum likelihood (PML), is to apply the EM method to each pair of variables in order to obtain estimates of the covariances. Each variance is typically estimated by applying the usual variance estimator to available data on that variable. Research has been conducted (a) to compare the CD method and various kinds of indirect methods for estimating Σ and the correlation matrix (Gleason & Staelin, 1975; Timm, 1970); (b) to compare the CD, direct, and indirect methods for estimating polychoric correlation matrices (Lee & Chiu, 1990); (c) in which V-C matrices were estimated using the CD, direct and indirect methods and subsequently either regression parameters (Little, 1988) or factor loadings (Brown, 1983; Finkbeiner, 1979) were estimated; and (d) in which both direct and indirect methods have been used to estimate the missing values and then, with the CD method also included, regression parameters estimated (Azen, van Guilder, & Hill, 1989). The methods were then compared in terms of their ability to (a) reconstruct the V-C matrix, (b) estimate the factor loadings, or (c) estimate the regression parameters.

The Problem

Methods for estimating Σ when some data are missing can be divided into three groups, the complete data method, the direct methods, and the indirect methods. In the complete data method, the complete observations are used to estimate Σ by least squares procedure. The methods in the second group incorporate an assumption that the observed data are drawn from a multivariate distribution of known form (usually the multivariate normal) but with unknown parameters. The parameters are estimated using the available data through an estimation procedure such as ML. Although the logic of the ML procedure is straightforward and the derived estimates have good properties such as consistency, sufficiency, efficiency, and asymptotic normality (Mood, Graybill, & Boes, 1974), the resulting computational procedures are generally complicated and time consuming. The complexity of the computation is often compounded by the presence of the missing values (Anderson, 1957; Hocking & Smith, 1968). The methods in the third group have no data distributional assumptions. The methods utilize the information in the observed values to construct the missing entries. Once the missing entries have been estimated, Σ or/and regression parameters are calculated through the LS procedure.

The problem in this study is threefold:

1. To compare the accuracy of estimation when the complete data only, direct, and indirect estimation methods are used in estimating missing values.

2. To compare the accuracy of estimation when V-C matrices are estimated by using the complete data only, direct, and indirect estimation methods.

3. To compare the accuracy of estimation when estimates of multiple linear regression parameters are calculated by LS after estimating the V-C matrix by the complete data only, direct and indirect methods.

Accuracy measures are presented in chapter 3.

Purpose of the Study

Among the direct V-C estimation methods, the study will investigate the direct ML through the Expectation-Maximization (EM) algorithm. The indirect methods to be studied are the mean replacement (MR), the complete pair variable only (PP), and the regression replacement (RG). The complete observations (CD) method will also be included.

The major purpose of the study is to compare the direct and indirect methods in their ability to estimate missing values, variance-covariance $\boldsymbol{\Sigma}$, and regression parameters when some data are missing in both independent and dependent variables. The study will include several factors: degree of skewness of the data (SK), average intercorrelation of the variables (R), percent of missing entries (PM), type of missing data (TM), residual variance (E), sample size (N), and number of variables (P).

Significance of the Study

The studies reported by Beale and Little (1975) and Little (1979) point to the superiority of the direct methods when regression parameters are calculated from estimated V-C matrices. Brown (1983) reported similar results for factor analysis. However, all three studies used only multivariate normal data. Azen, van Guilder, and Hill (1989) used estimated regression parameters to compare direct and indirect methods. Besides assuming that data were missing completely at random, Azen et al. (1989) also assumed that (a) the missing values occurred only in the independent variables, (b) the intercorrelations between any pair of variables were the same, and (c) the errors were normally distributed. To date, no study has looked at the characteristics of the estimated V-C matrix and regression parameters when (a) data from both independent and dependent variables are missing, (b) the intercorrelations between pairs of variables are not the same, and (c) the data are from a skewed distribution, a common characteristic of data from education research. The aim of this study is to meet these shortcomings in the literature. Of particular importance is the possibility that indirect methods may perform nearly as well as direct methods when the normality assumption is violated. Despite the superiority of the direct methods, researchers may tend to avoid them because of complexity.

Notation

The notation used is as follows:

1. Bold-faced capital letters designate matrices, for example **AxB** means multiplication of matrices **A** and **B**.

2. Bold-faced small letters designate vectors.

3. Letters in ordinary type designate a cell observation or a scalar depending on the text.

Furthermore, all abbreviations used are summarized in the Appendix, Table 28.

CHAPTER 2
REVIEW OF THE LITERATURE

Least Squares Estimation

Let **X** be an Nxp data matrix where x_{ij} is the value for observation i on variable j; i=1,...,N; j=1,...,p. Let $\mathbf{y}=\mathbf{x}_{p+1}$, such that $\mathbf{X}^{*}=(\mathbf{X},\mathbf{y})$, of dimension Nx(p+1), is the combined data set. Suppose

$$E(y|x_1,\ldots,x_p)=\beta_0+\sum_{j=1}^{p}\beta_j x_j \tag{1}$$

$$var(y|x_1,\ldots,x_p)=\sigma^2$$

and $x_1,\ldots,x_p,y$ have a joint distribution with mean $\boldsymbol{\mu}=(\mu_1,\ldots,\mu_p,\mu_y)$ and V-C matrix

$$\Sigma=\begin{bmatrix}\Sigma_{xx} & \Sigma_{xy}\\ \Sigma_{yx} & \sigma_{yy}\end{bmatrix} \tag{2}$$

where Σ_{xx} is the (pxp) V-C matrix of the independent variables (x_i, i=1,...,p), $\Sigma_{xy}=\Sigma_{yx}'$ is the (px1) column vector of covariances of the dependent variable (y) and each of the independent variables, and σ_{yy} is the variance of the dependent variable. Standard regression theory gives

$$\beta=\Sigma_{xx}^{-1}\Sigma_{yx}$$

$$\beta_0=\mu_y-\sum_{j=1}^{p}\beta_j\mu_j$$

and

$$\sigma^2=\sigma_{yy}-\Sigma_{yx}\Sigma_{xx}^{-1}\Sigma_{xy}$$

To estimate the parameters, the data are reduced to the sample means

$$\overline{\boldsymbol{x}}=(\overline{x}_1,\ldots,\overline{x}_p,\overline{y}=\overline{x}_{p+1})$$

and the sample V-C matrix $\mathbf{S}=(s_{jk})$

$$s_{jk}=(N-1)^{-1}\sum_{i=1}^{N}(x_{ij}-\overline{x}_j)(x_{ik}-\overline{x}_k)$$

In the absence of missing values, the multiple regression parameters can be estimated from the estimate of the mean vector and V-C matrix.

Maximum Likelihood Estimation

Let **X** (Nxp) be a data matrix representing an independently and identically distributed sample of N observations from the multivariate normal distribution with mean vector $\mu=(\mu_1,\ldots,\mu_p)$ and V-C $\Sigma=\{\sigma_{jk}, j=1,\ldots,p; k=1,\ldots,p\}$. The density of **X** is

$$f(\boldsymbol{X}|\mu,\Sigma)=(2\pi)^{-\frac{Np}{2}}|\Sigma|^{-\frac{N}{2}}\exp\left(-\frac{1}{2}\sum_{i=1}^{N}(\boldsymbol{x}_i-\mu)'\Sigma^{-1}(\boldsymbol{x}_i-\mu)\right)$$

where $|\Sigma|$ denotes the determinant of Σ. Thus, the likelihood $L(\Theta|X)$ of $\Theta=(\mu, \Sigma)$ is a function of μ and Σ for fixed observed X. The ML estimate of Θ is a value of Θ that maximizes $L(\Theta|X)$ or equivalently the natural logarithm of the likelihood, $\ln L(\Theta|X)=l(\Theta|X)$. For many models, the ML estimate is unique and furthermore the likelihood function itself is differentiable and bounded above. In such cases, the ML estimate can be found by differentiating the likelihood (or the loglikelihood) with respect to each component of Θ, setting the result equal to zero, and solving for Θ; that is

$$\partial/\partial\Theta_i\{l(\Theta|X)\}=0; \quad i=1,\ldots,k$$

where k is the number of components in Θ. Thus, the likelihood equation gives a set of k simultaneous equations, defined by differentiating $l(\Theta|X)$ with respect to all k components of Θ. For the multivariate normal sample, the ML estimates of μ and Σ are

$$\hat{\mu}=\bar{\boldsymbol{x}}=(\bar{x}_1, \ldots, \bar{x}_p)'$$

$$\hat{\Sigma}=\left(\frac{N-1}{N}\right)S=\frac{1}{N}\sum_{i=1}^{N}(\boldsymbol{x}_i-\bar{\boldsymbol{x}})(\boldsymbol{x}_i-\bar{\boldsymbol{x}})'$$

Let $g(\Theta)$ be a one-one function of the parameter Θ. Then the ML estimate of $g(\Theta)$ is $g(\hat{\Theta})$, the function evaluated at the ML estimate $\hat{\Theta}$ of Θ. The observed information matrix is defined by

$$I(\hat{\Theta}|X)=\partial^2/\partial\Theta_i^2\{l(\Theta|X)\}|_{\Theta=\hat{\Theta}}$$

and the standard errors of the estimates are given by the diagonal elements of $(I(\hat{\Theta}|\mathbf{X}))^{-1}$.

In the presence of missing values, the likelihood function will depend on whether the missing data mechanism can be ignored or not (Little & Rubin, 1987; Rubin, 1976). Little and Rubin (1987) said that if knowledge of the mechanism that leads to the missing data values does not influence the statistical model, then the mechanism is ignorable; otherwise the model will include the mechanism by including a distribution for the response indicator.

Distinction and Relevance of Missing Data Mechanism

In a single case i, i=1,...,N suppose $\mathbf{x}_i$ (1 x p) is a vector of p variables. Suppose $\mathbf{x}_i$ is partitioned such that $\mathbf{x}_i=(\mathbf{x}_{i0},\mathbf{x}_{im})$ where $\mathbf{x}_{i0}$ (1xk) is a vector of k (k≤p) observed variables, and $\mathbf{x}_{im}$ (1x(p-k)) is a vector of missing variables. Similarly, suppose $\mathbf{X}=(\mathbf{X}_0,\mathbf{X}_m)$, an Nxp data matrix has a density function $f(\mathbf{X}|\Theta)$, where Θ is the parameter vector of the data. The density function for $\mathbf{X}_0$ is

$$f(\mathbf{X}_0|\Theta)=\int f(\mathbf{X}_0,\mathbf{X}_m|\Theta)\,d\mathbf{X}_m$$

The likelihood of Θ based on $\mathbf{X}_0$ ignoring the missing data mechanism is a function proportional to $f(\mathbf{X}_0|\Theta)$:

$$L(\Theta|\mathbf{X}_0) \propto f(\mathbf{X}_0|\Theta)$$

Little and Rubin (1987) pointed out that knowledge of the mechanism that leads to the missing data values is a key element in choosing an appropriate analysis and in interpreting the results. Where the mechanism leading to the missing data does not enter explicitly in the data analysis, the mechanism is assumed ignorable and ML estimation is based on $L(\Theta|\mathbf{X}_0)$; otherwise the likelihood should include the missing data mechanism by including a distribution for the response indicator. Let $\mathbf{I}$ be the response indicator such that $I=1$ if an item is observed and $I=0$ otherwise. Thus the data consist of $(\mathbf{X}_0,\mathbf{I})$. Suppose the conditional distribution of $\mathbf{I}$ given $\mathbf{X}$ is indexed by an unknown parameter Ψ. By definition

$$f(\mathbf{X}_0,\mathbf{I}|\Theta,\Psi)=\int f(\mathbf{X}_0,\mathbf{X}_m|\Theta)f(\mathbf{I}|\mathbf{X}_0,\mathbf{X}_m,\Psi)d\mathbf{X}_m$$

The likelihood of the parameters is

$$L(\Theta,\Psi|\mathbf{X}_0,\mathbf{I}) \propto f(\mathbf{X}_0,\mathbf{I}|\Theta,\Psi) \qquad (3)$$

Rubin (1976) defined missing at random (MAR) and missing completely at random (MCAR) as follows: If the probability of response is independent of $\mathbf{X}_0$ and $\mathbf{X}_m$, then the missing data are MCAR. That is,

$$f(\mathbf{I}|\mathbf{X}_0,\mathbf{X}_m,\Psi)=f(\mathbf{I}|\Psi)$$

Thus, if the data are MCAR the likelihood of the parameters is

$$L(\Theta,\Psi|\mathbf{X}_0,\mathbf{I}) \propto f(\mathbf{X}_0|\Theta)f(\mathbf{I}|\Psi)$$

If the joint parameter space of (Θ,Ψ) is the product of the parameter space of each separately, that is the parameters Θ and Ψ are distinct, the likelihood of Θ is

$$L(\Theta|\mathbf{X}_0,\mathbf{I}) \propto f(\mathbf{X}_0|\Theta) \qquad (4)$$

Thus, the missing data mechanism is ignorable for likelihood-based inferences. The estimation of Θ would yield correct inferences from just the observed data (Rubin, 1976).

If the probability of response depends on $\mathbf{X}_0$ but not on $\mathbf{X}_m$, then the missing data are MAR. That is,

$$f(\mathbf{I}|\mathbf{X}_0,\mathbf{X}_m,\Psi)=f(\mathbf{I}|\mathbf{X}_0,\Psi)$$

Thus the likelihood of the parameters is

$$L(\Theta,\Psi|\mathbf{X}_0,\mathbf{I}) \propto f(\mathbf{X}_0|\Theta)f(\mathbf{I}|\mathbf{X}_0,\Psi)$$

In addition, if the parameters Θ and Ψ are distinct, then the likelihood of Θ is

$$L(\Theta|\mathbf{X}_0,\mathbf{I}) \propto f(\mathbf{X}_0|\Theta)$$

Thus, the missing data mechanism is ignorable for the likelihood-based inferences.

If the probability of response depends on $\mathbf{X}_m$, the missing data are neither MCAR nor MAR. The missing data mechanism is nonignorable.

If data are MAR or MCAR, the missing data mechanism can be ignored, and the likelihood function of Θ is based on only the observed data equation (4). If the missing data mechanism is nonignorable, then the likelihood function should include a missing data indicator, $\mathbf{I}$, variable. The likelihood function will be of the form in equation (3). When the data are MAR, the aim is to maximize the likelihood $L(\Theta|\mathbf{X}_0)$; otherwise $L(\Theta,\Psi|\mathbf{X}_0,\mathbf{I})$ is maximized.

Little and Rubin (1987) pointed out that for many patterns of incomplete data the likelihood function does not allow explicit ML estimates. In such cases, ML estimates can be obtained by iterative methods. One method that has come to common use is the EM algorithm.

The EM algorithm proposed by Dempster, Laird, and Rubin (1977) does not require second derivatives of the loglikelihood function to be calculated. The algorithm is an iterative method of maximum likelihood estimation that can be applied to any pattern of missing data (Dempster et al., 1977; Liski, 1985). Each iteration consists of an expectation step (E-step) and a maximization step (M-step). The algorithm begins with a starting value, $\Theta^{(0)}$ for Θ and replaces the complete data loglikelihood, $l(\Theta|\mathbf{X})$, with its expectation given the observed data and $\Theta^{(0)}$, $E[l(\Theta|\mathbf{X})|\mathbf{X}_0,\Theta^0]$. This is the E-step. The M-step involves the maximizing $E[l(\Theta|\mathbf{X})|\mathbf{X}_0,\Theta^{(0)}]$ to obtain a second estimate $\Theta^{(1)}$ of Θ. Each step of the algorithm increases the loglikelihood of Θ given $\mathbf{X}_0$. The procedure is repeated until convergence occurs.

Beale and Little (1975) have outlined the steps of finding the ML estimates of $\Theta=(\mu,\Sigma)$ through the EM algorithm. Let V_i be the set of variables observed in observation i, i=1,...,N. The estimation equations are

$$x_{ij}^{(t+1)}=E\left(x_{ij}\mid V_i;\mu^{(t)},\Sigma^{(t)}\right) \tag{5}$$

$$\mu_j^{(t)}=\frac{1}{N}\sum_{i=1}^{N}x_{ij}^{(t+1)} \tag{6}$$

$$\sigma_{jk}^{(t)}=\frac{1}{N}\sum_{i=1}^{N}\left(x_{ij}^{(t+1)}-\mu_j^{(t)}\right)\left(x_{ik}^{(t+1)}-\mu_k^{(t)}\right)+c_{ijk}^{(t)} \tag{7}$$

where

$$c_{ijk}^{(t)}=\mathrm{cov}(x_{ij},x_{ik}\mid V_i;\mu^{(t)},\Sigma^{(t)})$$

and $x_{ij}^{(t)}=x_{ij}$ if x_{ij} is observed,

= a linear combination of the variables in V_i, if x_{ij} is missing.

The equations (5) to (7) are iterated until a specified difference between successive iterations is met; the ML estimates are thus obtained.

The algorithm has slow convergence properties, particularly when the amount of missing data is relatively large. Furthermore, the algorithm does not provide standard errors for the ML estimates. However, it is insensitive to initial value; that is, it converges even if the initial value is poor.

Methods that Disregard the Incomplete Observations

There are three methods that estimate the variance-covariance matrix and the regression parameters without assuming an underlying data distribution and without estimating the missing entries.

Complete Data Only (CD) Method

Only cases which are complete in all p variables $\mathbf{x}_i=(x_{i1},\ldots,x_{ip})$ are used to estimate $\boldsymbol{\Sigma}$ and the regression parameters through least squares. Therefore, if x_{ij} and x_{ik}, $i=1,\ldots,N$, are any two components of $\mathbf{x}_i$, then for $j,k = 1,\ldots,p$

$$\bar{x}_j=\frac{\sum_{i=1}^{N} x_{ij}\lambda_i}{n_c}$$

$$\bar{x}_k=\frac{\sum_{i=1}^{N} x_{ik}\lambda_i}{n_c}$$

and

$$\hat{\sigma}_{jk}=\frac{\sum_{i=1}^{N} \lambda_i(x_{ij}-\bar{x}_j)(x_{ik}-\bar{x}_k)}{n_c-1}$$

where $\lambda_i=1$ if $\mathbf{x}_i$ is complete, otherwise $\lambda_i=0$, and $n_c=\Sigma_{i=1}^{N}\lambda_i$ is the number of complete cases.

Advantages of this approach are (a) simplicity, since standard complete-data statistical analyses can be applied without modifications, and (b) comparability of univariate statistics, since these are all calculated on a common set of cases. The major disadvantage is the potential loss of information in discarding incomplete cases. Suppose PM is the proportion of all the data in **X** that are missing. Gleason and Staelin (1975) have shown that the probability that any row of **X** will have no missing entry is approximately $(1-PM)^p$. Thus, as the number of variables, p, increases the proportion of rows with complete data decreases rapidly.

Complete Pair Variables Only (PP) Method

Glasser (1964) proposed a method that uses all available pairs of observations on each pair of variables to estimate **Σ**. Consequently, the method is sometimes known as Glasser method. Using the notation described in the CD method paragraph, for j, k =1,...,p,

$$\bar{x}_j = \frac{\sum_{i=1}^{N} x_{ij}\lambda_{i(jk)}}{n_{jk}}$$

$$\bar{x}_k = \frac{\sum_{i=1}^{N} x_{ik}\lambda_{i(jk)}}{n_{jk}}$$

and

$$\hat{\sigma}_{jk} = \frac{\sum_{i=1}^{N} \lambda_{i(jk)}(x_{ij}-\bar{x}_j)(x_{ik}-\bar{x}_k)}{n_{jk}-1}$$

where $\lambda_{i(jk)} = 1$ if x_{ij} and x_{ik} are present, otherwise $\lambda_{i(jk)} = 0$, and $n_{jk} = \Sigma_{i=1}{}^{N}\ \lambda_{i(jk)}$ is the number of cases with both x_j and x_k

present. The major disadvantage of the method is that the estimated V-C matrix is not necessarily positive definite. This problem is severe when the variables are highly correlated (Little, 1992). Haitovsky (1968) found that with highly correlated data, the PP method was inferior to the CD method. Kim and Curry (1977) found that the PP method was superior to the CD method when weakly correlated data were used. Azen et al. (1989) found that the PP regression estimates were inferior to the ML estimates for both normal and nonnormal regressors.

Allvalue Method

The Allvalue method (Dixon, 1983) is very similar to the PP method. The two differ only on the estimation of the mean of each variable. The Allvalue method estimates the mean of each variable using all observations for that variable and estimates covariances between pairs of variables using only cases complete in both variables. Using the notation described in the CD method paragraph, for j, k= 1,...,p the covariance between variable j and variable k is estimated as follows

$$\overline{x}_j = \frac{\sum_{i=1}^{N} x_{ij}\lambda_{i(jj)}}{n_{jj}}$$

$$\overline{x}_k = \frac{\sum_{i=1}^{N} x_{ik}\lambda_{i(kk)}}{n_{kk}}$$

and

$$\hat{\sigma}_{jk} = \frac{\sum_{i=1}^{N} \lambda_{i(jk)}(x_{ij}-\overline{x}_j)(x_{ik}-\overline{x}_k)}{n_{jk}-1}$$

where $\lambda_{i(jk)} = 1$ if x_{ij} and x_{ik} are present, otherwise $\lambda_{i(jk)} = 0$, and $n_{jk} = \Sigma_{i=1}^{N} \lambda_{i(jk)}$ is the number of cases with both x_j and x_k present.

Methods that Estimate the Missing Values First

Wilks' Method

Wilks (1932) proposed the mean replacement (MR) method: the estimate of the missing values on variable j is the jth-variable mean from all nonmissing values. Little (1992) called this method the unconditional mean imputation. In regression analysis, Afifi and Elashoff (1966) called such procedures, based on the substitution of means, zero order regression (ZOR) methods. Little and Rubin (1987) pointed out that earlier work also considered the case where the grand mean of all the available data was substituted for all missing values. Little and Rubin (1987) have shown that under MCAR the variance of the observed and imputed values for the jth variable is

$$\frac{s_{jj}(N_j-1)}{N-1}$$

where s_{jj} is the estimated variance from the N_j observed cases. Thus, the sample variance from the filled-in data set underestimates the variance by a factor of $(N_j-1)/(N-1)$. The sample covariance of $\mathbf{x}_j$ and $\mathbf{x}_k$ $(j<k=2,\ldots,p)$ from the filled-in data (Little & Rubin, 1987) is

$$\frac{s_{jk}(N_{jk}-1)}{N-1}$$

where

$$s_{jk}=\frac{1}{N_{jk}-1}\sum_{j,k}(x_{ij}-\overline{x}_j)(x_{ik}-\overline{x}_k)$$

and N_{jk} is the number of observed pairs. Again, the estimate from the filled-in data underestimates the magnitude of the covariance by a factor $(N_{jk}-1)/(N-1)$. Thus, inferences (tests and confidence intervals) may seriously be distorted by bias and overstated precision (Little, 1992).

The Classical Least Squares (CLS) Method

For the multiple regression model

$$y_i=\mu_y+\sum_{j=1}^{p}\beta_j(x_{ij}-\mu_j)+\epsilon_{ij}$$

Yates (1933) estimated μ_y, $\beta=\beta_1,\ldots,\beta_p$, and the missing data by minimizing

$$\sum_{i=1}^{N}\left(y_i-\mu_y-\sum_{j=1}^{p}\beta_j(x_{ij}-\overline{x}_{.j})\right)^2$$

where

$$\overline{x}_{.j}=\sum_{n_c}\frac{x_{ij}}{n_c}$$

n_c is the number of vector observations that are complete. That is, all observations for which values are missing are discarded and least squares regression performed on only the complete observations. The estimators of μ_y and β are the same as those in the CD method. The missing values are estimated by using the estimated model.

The Modified Zero-Order (MZOR) and the First-Order Regression (FOR) Methods

For the multiple linear regression analysis

$$y_i=\mu_y+\sum_{j=1}^{p}\beta_j(x_{ij}-\mu_j)+\epsilon_{ij};\, i=1,\ldots,N$$

the modified zero-order method (MZOR) involves minimizing

$$\sum_{i\notin I_i}\left(y_i-\mu_y-\sum_{j=1}^{p}(x_{ij}-\mu_j)\beta_j\right)^2+\sum_{i\in I_i}\left(y_i-\mu_y-\sum_{j=1}^{p}\gamma\beta_j\right)^2$$

(Hill & Ziemer, 1983), where $\gamma=\overline{x}_{jc}$ is the jth variable mean calculated using cases with complete data only and I_i represents the index set of cases with missing observation.

Hill and Ziemer (1983) presented the first-order regression (FOR) method which involves estimating p regressions

$$x_{ij}=\beta_{0j}+\sum_{k=1}^{p} x_{ik}\beta_{kj}+\epsilon_{ij};j=1,...,p;k\neq j$$

for complete observed cases only, where ϵ_{ij} is an error term. Least squares estimates of β_{kj} from complete sample observations are obtained. Then, each missing entry x_{ij} is replaced by its predicted value.

Zero-Order (ZOR) and First-Order (FOR) Combined Method

Jackson (1968) in a linear discriminant analysis study employed an iterative technique. The technique is to regress any variable in which missing observations occur on all remaining variables; means are substituted for all missing values for the initial iteration. The resulting regression equations are used to estimate the missing data. The procedure is iterated until estimates stabilize. Jackson (1968) found that this procedure was superior to the method of substituting means for the missing values.

Two-Stage Regression (TSR) Method

Afifi and Elashoff (1969a) gave a two-stage regression method for the simple linear regression method: First, the regression of x on y utilizing only the complete observations is estimated by least squares. From the estimated equation, the missing x observations are estimated. Second, the regression of y on x from the completed observations is estimated by least squares. From the estimated equation, the missing y observations are estimated.

Regression Replacement (RG) Method

Suppose $\mathbf{x}_i$, i=1,...,N, is partitioned by columns (variables) into two subvectors

$$\boldsymbol{x}=[\boldsymbol{x}_1\ \boldsymbol{x}_2]$$

where $\mathbf{x}_1$ of dimension 1xq (q<p) contains variables on which ith observation has missing data and $\mathbf{x}_2$ of dimension 1x(p-q) contains the variables on which ith observation has no missing data. Thus, an individual case is partitioned into an observed data set and a missing data set and $\mathbf{X}$ is partitioned conformably; that is,

$$\boldsymbol{X}=[\boldsymbol{X}_1\ \boldsymbol{X}_2] \tag{8}$$

Without loss of generality, assume the mean of $\mathbf{x}_j$ is zero and var($\mathbf{x}_j$)=1 for all j=1,..., p; then the correlation matrix, R, is

$$R=N^{-1}\boldsymbol{X}'\boldsymbol{X}=N^{-1}\begin{bmatrix}\boldsymbol{X}_1'\boldsymbol{X}_1 & \boldsymbol{X}_1'\boldsymbol{X}_2\\ \boldsymbol{X}_2'\boldsymbol{X}_1 & \boldsymbol{X}_2'\boldsymbol{X}_2\end{bmatrix}=\begin{bmatrix}R_{11} & R_{12}\\ R_{21} & R_{22}\end{bmatrix}$$

Gleason and Staelin (1975) gave the regression weights for estimating $\mathbf{X}_1$ from $\mathbf{X}_2$ as $\mathrm{R}_{22}^{-1}\mathrm{R}_{21}$. Hence,

$$\hat{X}_1 = X_2 R_{22}^{-1} R_{21} \tag{9}$$

The regression method requires first estimating **R** with the CD, MR, or PP method. The missing entries for any individual (row) of **X** are estimated by equation (9) with the nonmissing elements of the row becoming $\mathbf{X}_2$ and the matrices $\mathbf{R}_{22}$ and $\mathbf{R}_{21}$ being drawn from the relevant portions of the estimated correlation matrix. When the CD method is used to estimate **R**, the RG method is called Bucks method. Walsh (1961) generalized Buck's method by computing all possible linear regressions of the variable j with a missing value on variable k for $j \neq k$, $j,k=1,\ldots,p$, for all complete pairs. A weighted average of the estimated values was proposed to fill the missing data.

Combining Buck's and Walsh's ideas, Afifi and Elashoff (1966) proposed use of all possible multiple regressions for the estimation of the missing values. However, this technique is not computationally feasible when p is large and a large proportion of the data is missing. Buck's and Walsh's methods are examples of what Afifi and Elashoff (1967) called first-order methods.

Principal Components (PC) Method

Wold (1966) proposed a missing value estimation method that utilizes the principal components from an incomplete data matrix. Suppose the first n ($n<p$) components capture

most of the error-free information, while the last p-n components reflect mostly the effects of errors of measurement. The method is sensitive to the value of n considered in the estimation. The theory behind this method is given by Gleason and Staelin (1975). Applying the Singular-Value Decomposition theorem, **X** can be written in the form

$$X=UD^{\frac{1}{2}}V'$$

where **D** is a diagonal matrix of eigenvalues of the correlation matrix **R** of the variables of **X**, **V** is the matrix of eigenvectors of **R**, and $\mathbf{U} = \mathbf{XVD}^{-1/2}$. Suppose the largest n eigenvalues of **R** are retained. Let $\mathbf{D}_n$ be the (nxn) submatrix of **D** which contains the n eigenvalues and $\mathbf{V}_n$ the matrix of corresponding eigenvectors. Then,

$$\hat{X}=XV_nV_n'$$

of rank n (Gleason & Staelin, 1975). Suppose **X** is partitioned as in equation (8); and correspondingly, $\mathbf{V}_n$ is partioned as $\mathbf{V}_n = [\mathbf{V}_1\ \mathbf{V}_2]'$, where $\mathbf{V}_1$ is qxn and $\mathbf{V}_2$ is (p-q)xn. Let,

$$W=V_nV_n'=\begin{bmatrix}V_1V_1' & V_1V_2'\\ V_2V_1' & V_2V_2'\end{bmatrix}=\begin{bmatrix}W_{11} & W_{12}\\ W_{21} & W_{22}\end{bmatrix}$$

Gleason and Staelin (1975) have shown that

$$\hat{X}_1 = X_2 W_{21}(I - W_{11})^{-1} \tag{10}$$

This is the principal components method for reconstructing $\mathbf{X}_1$ from $\mathbf{X}_2$. The Dear method (Gleason & Staelin, 1975) uses only the first principal component and is thus a special case of the principal component method. Gleason and Staelin (1975) showed that the covariance matrix of $\mathbf{X}_1$ with $\hat{\mathbf{X}}_1$ is

$$N^{-1} X_1' \hat{X}_1 = R_{11} - R_{11}^{*}(I - W_{11})^{-1} \tag{11}$$

where

$$R^{*} = \begin{bmatrix} R_{11}^{*} & R_{12}^{*} \\ R_{21}^{*} & R_{22}^{*} \end{bmatrix} = R - V_n D V_n'$$

is the residual matrix, which is obtained by removing that portion of $\mathbf{R}$ which is accounted for by $\mathbf{V}_n$ and $\mathbf{D}_n$. The matrix $\mathbf{R}^*$ depends on the p-n smallest eigenvalues of $\mathbf{R}$. From equation (11), if $\mathbf{X}$ is of rank n or less, then $\hat{\mathbf{X}}_1$ reproduces $\mathbf{X}_1$ exactly provided (a) $n<p-q$ or $q<p/2$ and (b) $\mathbf{I}-\mathbf{W}_{11}$ is nonsingular (Gleason & Staelin, 1975).

Gleason and Staelin, (1975) studied the relationship between the RG and PC methods and showed that

$$W_{21}(I - W_{11})^{-1} = R_{22}^{-1} R_{21} - R_{22}^{-1} R_{21}^{*}(I - W_{11})^{-1} \tag{12}$$

The left-hand side of equation (12) is the transformation used by the Principal Components method, equation (10). The right-hand side is the transformation used by the Regression method, equation (9), but reduced by a term that depends on the smallest p-n eigenvalues of $\mathbf{R}$. Thus, theoretically the

Principal Components method and the Regression method are related as follows (Gleason & Staelin, 1975):

1. If the rank of **X** exceeds n, then $\mathbf{R}^*$ will be a nonzero matrix. Thus, the Principal Components method will differ from the Regression method.

2. If the rank of **X** is less than or equal to n, then $\mathbf{R}^*$ and thus $\mathbf{R}^*_{21}$ will be negligible or nearly zero matrix. Thus, the two methods should perform similarly.

When Is the CD Method Sufficient?

Hill and Ziemer (1983) have proposed a test statistic for checking the adequacy of the CD method in the estimation of regression parameters when some data are missing. Let **Y** and **X** be partitioned as follows:

$$\begin{bmatrix} Y_c \\ Y_k \end{bmatrix} : \begin{bmatrix} X_c \\ X_k \end{bmatrix}$$

where $\mathbf{X}_k$ (Jxp) has at least one variable missing in each row and $\mathbf{Y}_k$ (Jx1) is the corresponding observed response vector, $\mathbf{X}_c$ (nxp) and $\mathbf{Y}_c$ (nx1) are the complete data, n=N-J. For the complete sample

$$Y_c = X_c\beta + e_c \tag{13}$$

where $\mathbf{e}_c$ is an iid random variable with mean 0 and variance σ^2. The LS estimator for the complete sample can be shown to be

$$\beta_{CD} = (\mathbf{X}_c{}'\mathbf{X}_c)^{-1}\mathbf{X}_c{}'\mathbf{Y}_c$$

$$\mathrm{var}(\beta_{CD}) = \sigma^2 (\mathbf{X}_c' \mathbf{X}_c)^{-1}$$

$$\beta_{CD} \sim N(\beta, \sigma^2 (\mathbf{X}_c' \mathbf{X}_c)^{-1})$$

For the incomplete sample

$$\mathbf{Y}_k = \mathbf{X}_k \beta + \mathbf{e}_k$$

where $\mathbf{e}_k$ is an iid random variable with mean 0 and variance σ^2. Suppose the missing entries are estimated and a (Jxp) matrix $\mathbf{X}_m$ is substituted for the incomplete $\mathbf{X}_k$ so that

$$\begin{aligned}\mathbf{Y}_k &= \mathbf{X}_k \beta + \mathbf{X}_m \beta - \mathbf{X}_m \beta + \mathbf{e}_k \\ &= \mathbf{X}_m \beta + (\mathbf{X}_k - \mathbf{X}_m) \beta + \mathbf{e}_k \\ &= \mathbf{X}_m \beta + \delta + \mathbf{e}_k \\ &= \mathbf{X}_m \beta + \mathbf{v} \qquad (14)\end{aligned}$$

where $\mathbf{v} = \delta + \mathbf{e}_k$ and $E(\mathbf{v}) = \delta$. Hill and Ziemer (1983) showed that the estimator of β that combines (13) and (14) is

$$\beta = (X_c'X_c + X_m'X_m)^{-1}(X_c'Y_c + X_m'Y_k)$$

They also showed that

$$var(\beta) = \sigma^2 (X_c'X_c + X_m'X_m)^{-1}$$

and that the matrix

$$var(\beta_{CD}) - var(\beta) = \sigma^2 (X_c'X_c)^{-1} X_m' (X_m (X_c'X_c)^{-1} X_m' + I)^{-1} X_m (X_c'X_c)^{-1} \qquad (15)$$

is positive semidefinite. Furthermore, they showed that $\hat{\beta}$ was biased, while β_{CD} was not. The choice of β_{CD} over $\hat{\beta}$ depended on the difference of the mean squared errors for the parameter estimates (Hill & Ziemer, 1983)

$$MSE(\beta_{CD})-MSE(\beta)=\Delta$$

where

$$\mathrm{MSE}(\beta_{CD})=\sigma^2(\mathbf{X}_c{}'\mathbf{X}_c)^{-1}$$

and

$$MSE(\beta)=\sigma^2\left(X_c'X_c+X_m'X_m\right)^{-1}+\left(X_c'X_c+X_m'X_m\right)^{-1}X_m'\delta\delta'X_m\left(X_c'X_c+X_m'X_m\right)^{-1}$$

Thus

$$\Delta=var(\beta_{CD})-var(\beta)-(Bias(\beta))^2$$

is given as

$$\Delta=(\mathbf{X}_c{}'\mathbf{X}_c+\mathbf{X}_m{}'\mathbf{X}_m)^{-1}\mathbf{X}_m{}'[\sigma^2(\mathbf{X}_m(\mathbf{X}_c{}'\mathbf{X}_c)^{-1}\mathbf{X}_m{}'+\mathbf{I})-\delta\delta']\mathbf{X}_m(\mathbf{X}_c{}'\mathbf{X}_c+\mathbf{X}_m{}'\mathbf{X}_m)^{-1}$$

which was shown to be positive definite if

$$\lambda=\frac{\delta'\left(X_m\left(X_c'X_c\right)^{-1}X_m'+I\right)^{-1}\delta}{2\sigma^2}\leq\frac{1}{2}$$

where λ is the non-centrality parameter of the test statistic

$$\gamma=\frac{(Y_k-X_m\beta_{CD})'\left(X_m\left(X_c'X_c\right)^{-1}X_m'+I\right)^{-1}(Y_k-X_m\beta_{CD})}{J\hat{\sigma}^2}$$

which is reported to be distributed as an $F(J,n-p,\lambda)$ if the null hypothesis

$$H_0:\lambda\leq 1/2$$

is true, where

$$\hat{\sigma}^2 = \frac{Y_c'\left(I - X_c\left(X_c'X_c\right)^{-1}X_c'\right)Y_c}{N-p}$$

From this analytic approach, Hill and Ziemer (1983) concluded that the CD method will give good estimates if H_0 is rejected; otherwise, methods that estimate missing values first should be used.

Evaluation of the Estimation Methods

Criteria for (a) the comparison of estimated missing values to observed values, (b) the comparison of estimated V-C matrix to Σ, (c) the comparison of estimated V-C matrices from two estimation methods, (d) the comparison of estimated multiple regression parameters to those obtained from the complete full data, and (e) the comparison of the multiple regression model fit are required for an effective evaluation of the estimation methods.

Evaluation Based on the Estimation of the V-C Matrix

Timm (1970) found that criteria based on determinants $|\Sigma-\hat{\Sigma}_\alpha|$ for comparing Σ to $\hat{\Sigma}_\alpha$ or $|\hat{\Sigma}_\alpha|/|\hat{\Sigma}_\beta|$ for comparing $\hat{\Sigma}_\alpha$ and $\hat{\Sigma}_\beta$, where α and β are any of the missing data estimation methods, are heavily biased by small eigenvalues since the determinant of a matrix is the product of its eigenvalues. Thus, such criteria are inappropriate. Timm (1970) instead proposed the Euclidean norm,

$$\|\Sigma - \hat{\Sigma}_\alpha\|^2 = tr((\Sigma - \hat{\Sigma}_\alpha)(\Sigma - \hat{\Sigma}_\alpha)') \qquad (16)$$

a measure of closeness between the estimated and the true V-C matrices. The major drawback with the use of the Euclidean norm is that its distribution is unknown and thus one cannot formulate a statistical significance test. Timm (1970) suggested the following rule: choose the technique which minimizes $E\{\|\Sigma - \hat{\Sigma}_\alpha\|^2\}$. He further suggested the ratio of the Euclidean norms as a measure of the overall comparative efficiency of any pair estimation methods. The efficiency of estimation method α relative to method β is defined as

$$eff\left(\frac{\hat{\Sigma}_\alpha}{\hat{\Sigma}_\beta}\right) = \frac{\|\Sigma - \hat{\Sigma}_\alpha\|^2}{\|\Sigma - \hat{\Sigma}_\beta\|^2} \qquad (17)$$

Estimation method α is preferable if the ratio is less than one, otherwise method β is the best.

Gleason and Staelin (1975) modified the Euclidean norm statistic and applied it to the correlation matrix. Their statistic is defined as

$$D_\alpha = \left(\frac{\|R - \hat{R}_\alpha\|^2}{p(p-1)}\right)^{\frac{1}{2}} \qquad (18)$$

representing the root-mean-square deviation of estimated versus actual values for the off-diagonal correlations.

Another way of comparing the estimation methods, is to construct a measure of the quality of the estimated missing entry. Let x_{ij} be the value in a complete data matrix that corresponds to a missing entry in an incomplete matrix. Suppose the variance of variable j in the complete data matrix is σ_j^2. Let $x_{ij}^{(\alpha)}$ be the estimate of x_{ij} derived by method α. Let K denote the set of entries missing from the matrix. Gleason and Staelin (1975) proposed the following statistic

$$Q_\alpha = \sum_{i,j \epsilon K} \frac{\left(x_{ij}^{(\alpha)} - x_{ij}\right)^2}{\sigma_j^2 Np(PM)} \tag{19}$$

an index measuring the reconstruction of the elements in K, where PM is the proportion of missing values. The smaller Q_α, the better the fit. Therefore, the interest is to find an estimation method α which minimizes Q_α.

Evaluation Based on the Multiple Regression Model Fit

Let $\hat{\beta}_{j\alpha}$, j=0,1,...,p, denote the multiple regression coefficients estimated from incomplete data by method α. The estimated coefficient can be compared to the true coefficient by a t-test (Little, 1979)

$$t_j = \frac{\beta_j - \beta_{j\alpha}}{SE(\beta_{j\alpha})}$$

which is distributed as a t with (N-p+1) degrees of freedom.

Beale and Little (1975) recommended judging an estimation method by the overall success of the regression equation rather than by closeness of individual regression coefficients to their true values. Thus, the criterion Beale and Little (1975) proposed was the residual sum of squares of deviations of the observed and estimated values of the dependent variable when the deleted values were restored, defined as

$$S_\alpha=\sum_{i=1}^{n_c}\left(y_i-\hat{\beta}_{0\alpha}-\sum_{j=1}^{p}\hat{\beta}_{j\alpha}x_{ij}\right)^2 \tag{20}$$

where, $\hat{\beta}_{0\alpha}$ and $\hat{\beta}_{j\alpha}$, j=1,...,p, are the regression coefficients estimated by using method α, and x_{ij} and y_i are the values of all variables from the complete data only; and n_c is the number of complete data observations. The smaller the value of S_α, the better the estimation method α.

Donner and Rosner (1982) computed

$$d_{i\alpha}=|\hat{\beta}_{i\gamma}-\beta_i|-|\hat{\beta}_{i\alpha}-\beta_i| \tag{21}$$

where $\hat{\beta}_{i\alpha}$ and $\hat{\beta}_{i\gamma}$ are the ith, i=0,1,...,p regression coefficient from methods α and γ respectively, and β_i is the true regression coefficient. Then, Donner and Rosner (1982) used $d_{i\alpha}>0$ as a measure of relative accuracy of estimator $\hat{\beta}_{i\alpha}$ to $\hat{\beta}_{i\gamma}$.

Donner and Rosner (1982) also used the relative squared errors of the regression coefficients as a criterion, defined as

$$RE_{i\alpha}=\frac{MSE(\beta_{i\gamma})}{MSE(\hat{\beta}_{i\alpha})} \tag{22}$$

i=0,1,...,p. If $RE_{i\alpha}<1$, Donner and Rosner regarded method γ as better than method α for estimating β_i .

Azen et al. (1989) used two criteria for the assessment of the estimation methods:

1. A measure of how well the estimation method estimated the regression coefficients, defined as

$$C_1=\sum_{i=0}^{p}\frac{(\beta_i-\hat{\beta}_i)^2}{W} \tag{23}$$

where

$$W=E\left(\sum_{i=0}^{p}(\beta_i-\hat{\beta}_i)^2\right)$$

given no missing data, and p is the number of independent variables.

2. A measure of how well the missing values are estimated, defined as

$$C_2=\frac{1}{3N(PM)}\sum_i\sum_j\frac{d_{ij}^2}{\min\, d_{ij}^2} \tag{24}$$

where the summation is over all missing values, PM is the overall proportion of missing data among all x_{ij}s,

$$d_{ij}^2=\min\, d_{ij}^2+\left(x_{ij}-x_{ij}^*\right)$$

x^*_{ij} is the estimated missing value, x_{ij} is the true value, and

$$\min d_{ij}^2 = \text{var}(\mathbf{x}_j) - \mathbf{C}_{xw}\mathbf{C}_{ww}^{-1}\mathbf{C}_{wx}$$

where $\mathbf{C}_{xw}$ is the estimated covariance of the jth variable, $\mathbf{x}_j$ with $\mathbf{w}$ (the vector of observed variables for case i with y_i included), and $\mathbf{C}_{ww}$ is the estimated dispersion matrix for $\mathbf{w}_i$. Then, two-way analyses of variance of the C_1 and C_2 values were conducted to compare the estimation methods.

Comparison of the Estimation Methods from Past Research

The comprehensive review by Afifi and Elashoff (1966) points out that most of the pioneering work dealing with missing data techniques in the estimation of $\mathbf{\Sigma}$ assumed that the data were from a multivariate normal distribution and proceeded to obtain ML estimates for the parameters. Comparative studies among the indirect methods have been conducted (Gleason & Staelin, 1975; Timm, 1970). However, similar studies involving either direct methods or direct and indirect methods are very scarce and incomplete. In this section a review of what has been done so far is presented.

Comparison Based on V-C Estimation

Timm (1970) compared the CD, MR, Dear, and Buck methods. All these methods are indirect methods. The comparison of the methods was based on the accuracy with which $\mathbf{\Sigma}$ and $\mathbf{R}$ were estimated. From educational and psychological research, a number of V-C matrices with p=2, 5 or 10, and average intercorrelation ranging from 0.2 to 0.8

were selected. Complete data matrices were generated by using the Kaiser and Dickman (1962) procedure. Also, sample size (N=50,100,200), and percent of missing data (PM=1,10,20) were varied. Timm (1970) used the Kaiser (1968) definition of average intercorrelation

$$R=\frac{\lambda_1-1}{p-1} \tag{25}$$

where λ_1 is the largest eigenvalue of **R**. From each of the complete data matrices a fixed proportion of entries were deleted at random. Then each of the estimation methods was used to reconstruct the complete data matrix, and its associated V-C and **R** matrices. No uniformly best estimating method for **Σ** was found; however, based on efficiency indices (equation 17), along with the variation in the Euclidean norm, Timm (1970) found that

1. The performance of the various estimation methods depended on whether **Σ** or **R** was being estimated. Different conclusions were obtained when **Σ** was estimated and when **R** was estimated.

2. With weakly correlated observations, that is $R<0.3$, (a) both the Buck and the Dear methods were recommended for $p\geq5$, $N\geq100$, and $PM\geq1$ for the estimation of **Σ**; (b) avoiding all methods was recommended if the conditions in (a) were not obtained; (c) the Buck and Dear methods worked equally well for $p\geq5$, $N\geq100$, and $PM\leq10$ for the estimation of **R**; and

(d) the Dear method was recommended when PM=20, otherwise the MR was recommended.

3. With intermediate correlations, that is $0.4<R<0.6$, (a) the Buck method was recommended for $p\geq5$, $N\geq50$, and $PM\geq1$; (b) the Dear method was recommended for $p=2$, $N\geq50$, and $PM\geq1$ or for $p\geq5$ and PM=20. Recommendations (a) and (b) were made for the estimation of $\mathbf{\Sigma}$ and the estimation of $\mathbf{R}$.

4. With highly correlated observations, that is $R\geq0.8$, the Buck method was recommended for $p\geq5$, $N\geq50$, and $PM\leq1$, for the estimation of $\mathbf{R}$, otherwise the Dear method was recommended.

5. In the estimation of $\mathbf{R}$, Timm (1970) noted that both the MR and CD methods tended to reduce the average intercorrelation of variables, while the Dear method increased it. The Buck method reduced it only under low intercorrelation.

Gleason and Staelin (1975) compared the PC, Dear, RG, PP, and MR methods in terms of ability to estimate the $\mathbf{R}$, as well as in terms of ability to estimate missing entries. Gleason and Staelin (1975) generated complete data matrices for combinations of sample size (N=50, 100, 200), number of variables (p=10, 15, 30), and average intercorrelation (R=0.1 to 0.6). By varying percent of missing data (PM=4, 12, 20), three incomplete data matrices were constructed by random deletion from each complete data matrix. Then, each of the estimation methods was used to estimate the missing data and an estimate of **R** was obtained. Gleason and Staelin (1975) defined the average intercorrelation as

$$R=\left(\frac{\|R\|^2-p}{p(p-1)}\right)^{\frac{1}{2}} \tag{26}$$

where

$$\|R\|^2=\sum_{i=1}^{p}\lambda_i^2$$

and λ_i, i=1, . . ., p are the eigenvalues of **R**. Euclidean norm statistics (D_α) defined as in (18) with **R** matrices, and the index of missing data reconstruction (Q_α) defined as in (19) were regressed against N, p, PM, R using multiplicative regression models of the form

$$z=\beta_0(1-R)^{\beta_1}N^{\beta_2}p^{\beta_3}(PM)^{\beta_4} \tag{27}$$

or

$$z=\beta_0 R^{\beta_1}N^{\beta_2}p^{\beta_3}(PM)^{\beta_4} \tag{28}$$

where z is either D_α or Q_α. The use of either (27) or (28) was justified because theoretically the following conditions hold:

(i) $D_{MR} \rightarrow 0$ as $R \rightarrow 0$

(ii) $D_{PP} \rightarrow 0$ as $R \rightarrow 1$

(iii) $D_{MR} \rightarrow 0$, $D_{PP} \rightarrow 0$ as $PM \rightarrow 0$

Thus, equation (28) was applied to the MR method, while equation (27) was applied to the PP, PC and RG methods. The comparisons of the estimation methods were based on these regression models. Gleason and Staelin (1975) found that

1. The choice of the estimation method for **R** was strongly influenced by N and R. When there is little intercorrelation in the data and few observations, the MR method is best. For large correlations and a large number of observations, the PP method performed better and the RG method and the PC method performed much better than the MR method. For example, the PP method was better than the MR method for N=50, 100, 200 when R>0.25, 0.20, 0.15 respectively.

2. For values of R>0.2, the PC and RG methods gave estimates of the missing data that were superior to those given by the MR method. In addition, the relative gains of the PC and RG methods increased rapidly as R increased, irrespective of the levels of PM and N.

3. For R<0.2, the MR method was best since there was insufficient redundancy in the data to provide any better estimates of the missing entries through the PC or RG methods than was given by the mean of the variable. This result was true irrespective of the levels of PM and N.

4. The PC method was computationally faster than the RG method; but the two methods were very similar when their Q_α and D_α curves were studied.

5. No broad conclusions were given in relation to either the percent missing data or the number of variables.

However, Gleason and Staelin (1975) acknowledged that the best estimation method is a function of PM and p as well.

Lee and Chiu (1990) compared the ML and PML direct methods and the CD, MR, RG, and PC indirect methods for estimating a polychoric correlation when data are missing. Discrete trivariate polytomous observations were obtained by transforming trivariate normal variables using preassigned thresholds. Some of the observations on the three variables were purposely removed at random. For the MR, RG, and PC methods completed data sets were obtained and the parameter estimates were obtained by the ordinary ML (OML) method for estimating polychoric correlations. For the CD method OML for estimating polychoric correlations was applied to the complete data. Also, from the initial data before missing entries were created, OML estimates were obtained. Lee and Chiu (1990) found that

1. The OML, direct ML and PML estimates of the polychoric correlation matrix were very close to each other, and the ML and PML estimates were generally better than the estimates produced by the CD, MR, RG and PC methods. These results were more pronounced for asymmetric data than for symmetric data. Lee and Chiu (1990) did not work with data from specific known distributions; but rather, they used data distribution pattern to classify data as symmetric or asymmetric. However, the initial correlation matrix used in the RG and PC methods was calculated using CD, which may

have affected the results. Gleason and Staelin (1975), found that the initial **R** has a strong influence on the performance of the RG and PC methods.

2. The direct ML and PML estimates were not as strongly affected by the asymmetry of the data as were the indirect methods.

3. For the direct ML and PML methods, the root mean squared error associated with larger population correlations was smaller than that associated with smaller population correlations.

4. The estimates obtained by using the MR, RG, and PC methods were very close to each other. The CD method gave better estimates than the MR, RG and PC methods. Theoretically, the CD method ought to perform less well than the other methods because as noted earlier, as the number of variables (p) increases, the proportion of rows with complete data decreases rapidly.

5. The PML method was comparable with the direct ML method in terms of accuracy. However, the direct ML was recommended because it resulted in accurate estimates with moderate sample sizes even when there were many missing observations.

Comparisons Based on Regression Model Fit

Afifi and Elashoff (1967) studied the CD, MR, and MZOR methods for estimating β in the simple linear regression model when the missing **Y** and **X** observations occured at random.

From the estimators of the βs, Afifi and Elashoff (1967) found that

1. For fixed ρ (correlation of y and x), m_x/N, m_y/N, where m_x and m_y are the number of missing observations on x and y, the efficiencies of the MR and MZOR coefficient estimators relative to that of the CD estimators decreased as N increased. They defined the efficiency of method α regression coefficient estimator relative to method γ estimator as $MSE(\hat{\beta}_\gamma)/MSE(\hat{\beta}_\alpha)$.

2. For fixed N, m_x, m_y these efficiencies decreased as ρ increased.

3. If $|\rho| \leq 0.30$ and $N \leq 70$ the MR and MZOR estimators were better than the CD.

4. For fixed N and m_y/N, the efficiencies of the MR and MZOR estimators relative to the CD estimator increased as m_y/N increased for $|\rho| \leq 0.30$. The same was true if the roles of x and y were exchanged.

Afifi and Elashoff (1969a) studied the FOR, the TSR, and ML methods, and all the methods studied by Afifi and Elashoff (1967). Again, the simple linear regression parameters β, and σ^2 were studied. Though no assumption of the bivariate normality for x and y was required to define the estimators, the assumption was essential for the derivation of the asymptotic distributions of the parameters.

Let p_x and p_y be the proportion of the x and y observations missing, respectively. For the estimators of β, Afifi and Elashoff (1969a) found that

1. The TSR estimator of β was better than all other estimator and comparable to the ML estimator, for all ρ, p_x, and p_y.

2. For fixed $|\rho|$ and p_x, the asymptotic efficiency of the TSR β estimator increased as p_y increased. The same was true if the roles of x and y were interchanged.

3. Based on finite sample efficiencies, the TSR estimator of β was comparable to the ML estimator unless $|\rho|\leq 0.1$ for all $N\geq 200$. If $|\rho|\leq 0.1$, the ZOR estimator of β was better than all other estimators for $N\leq 300$.

For the comparisons among the estimators of σ^2, Afifi and Elashoff (1969a) found that

1. The TSR estimator was better than the rest, except the ML estimator, for all values of ρ if $p_x\leq 0.4$ and $p_y\leq 0.4$.

2. Based on asymptotic efficiency, the TSR estimator and ML estimator were comparable.

Afifi and Elashoff (1969b) in a follow-up note, compared the small sample performance of estimators estimated by the FOR and the TSR methods. Bias and efficiency of the estimators of β and σ^2 were studied. It was found that

1. When N=20 and $p_x=p_y=0.4$, the TSR regression method generally had smaller bias than the other methods.

2. The ranking of the estimators based on small sample efficiency agreed with the ranking based on Afifi and Elashoff (1969a) asymptotic efficiency.

Beale and Little (1975) compared six methods for estimating β when data are missing at random. Only the multivariate normal case was investigated. Data values from both the independent and dependent variables were missing at random. The six methods were

1. Ordinary least squares on complete observations only.

2. Buck's method.

3. Iterated Buck's method, which gives ML estimates.

4. Ordinary least squares with **Y** present after fitting missing values of the independent variables by ML using independent variables only.

5. The method in 4, but with missing values given fractional weights.

6. The method in 5, but using a covariance matrix for all the variables found by method 3, to find the fitted values and estimate the weights.

The residual sum of squares of deviations of the observed and fitted values of the dependent variable as defined in (20) was used as a criterion for judging the effectiveness of each method. The study was repeated ten times. Overall, Method 3 was superior to all other methods. Method 1 was even inferior to Method 2. Method 5, an improvement over Method 4 in some situations, was inferior to Method 1. Method 6, an improvement over Method 5, did quite well but was still inferior to Method 3.

Little (1979) studied four methods for estimating the multiple regression parameters, β, and σ^2 when some data are missing at random. Only the multivariate normal distribution was investigated. Normally distributed data of four variables $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$, and $\mathbf{y}$ for five problems were generated of the form

$$y_k=x_1+\frac{1}{2}x_2+\frac{2}{5}x_3+\epsilon_k$$

where $\epsilon_k \sim N(0,\sigma_y^2)$ and $(x_1,x_2,x_3)' \sim MN(0,\Sigma)$. Six deletion patterns were used to generate missing values. However, only the independent variables were allowed to have missing values. For each problem and deletion pattern 50 data sets were generated. The four estimation methods studied were:

1. Ordinary least squares on the data before deletion.

2. Ordinary least squares on the complete observations only.

3. ML estimation via the Hartley and Hocking (1971) method of scoring.

4. ML estimation via the EM algorithm.

For each method, the V-C matrix and the regression parameters were obtained. Little (1979) compared the four methods under six deletion patterns by analyzing (a) the pivotals of the regression parameters, defined as

$$PQ(\beta_j) = \frac{\hat{\beta}_j - \beta_j}{SE(\hat{\beta}_j)} \tag{29}$$

and (b) the mean squared deviation of $\hat{\beta}_j$ from its true value denoted by MSE($\hat{\beta}_j$). For each problem and deletion pattern, 50 data sets were generated. From the pivotals, the means (MPQ) and mean sum of squares (MSPQ) were calculated, and two-way analysis of variance on MPQ and MSPQ values was conducted with the six deletion patterns and five problems as factors. The analysis of the MSPQ showed that

1. The distribution of $\hat{\beta}_j$ is not normal for methods 2, 3 and 4.

2. The accuracy of the normal approximation was less satisfactory for the severely deleted patterns than for the less severely deleted patterns.

3. When deviations of the MPQ from zero were studied, methods 3 and 4 had slight positive bias.

With MSE($\hat{\beta}_j$) values, Little (1979) found that

1. For the severely deleted patterns, the relative efficiency of method 1 to either method 3 or method 4, defined as the ratio of the two MSE($\hat{\beta}_j$), was very low.

2. Methods 3 and 4 were more efficient than Method 2.

Donner and Rosner (1982) studied four missing data estimation methods applied to multiple regression with two independent variables,

$$y_i=\beta_0+\beta_1 x_{1i}+\beta_2 x_{2i}+e_i; \quad i=1,\ldots,N$$

where the e_is are iid with mean zero and variance σ^2. The four methods were the CD, PP, ML and linear prediction methods. Missing values, for PM=10, 25, and 50, occurred at random on x_2 only. The correlation of y and x_1 was fixed at 0.2. The correlations of y and x_2, and x_1 and x_2 were 0.2, 0.5 or 0.8. For each combination of ρ_{yx1}, ρ_{yx2}, ρ_{x1x2} and PM, N=100 pairs (x_1, x_2) were generated from a bivariate normal distribution with $\mu_1 = \mu_2 = 0$, $\sigma_1^2 = \sigma_2^2 = 1$.

Donner and Rosner derived the MSEs of the regression coefficients and noted that

1. The MSE of $\hat{\beta}_1$ for the PP method relative to the MSE of $\hat{\beta}_1$ for the CD method was high when the sample correlation (r) between x_1 and x_2 was small.

2. For estimating β_1, the accuracy of the linear prediction method relative to the accuracy of the CD method was high for large r.

For estimation of β_2, Donner and Rosner showed analytically that MSE($\hat{\beta}_2$) for the PP method was greater than MSE($\hat{\beta}_2$) for the CD method. Also, for estimation of β_2, accuracy of the linear prediction method and the CD method were equivalent.

To compare estimators in their monte carlo study, Donner and Rosner (1982) used the following criteria:

1. A difference criterion: $d_{ij}=|\hat{\beta}_{im}-\beta_i|-|\hat{\beta}_{ij}-\beta_i|$, i=1,2; j=c,p,l denotes the CD, PP and linear prediction methods respectively, and m denotes the ML method. The proportion of 100 replications with $d_{ij}>0$ was a measure of relative accuracy of the estimator $\hat{\beta}_{ij}$ to $\hat{\beta}_{im}$.

2. The relative mean squared errors

$$RE_{ij}=\frac{MSE(\beta_{im})}{MSE(\beta_{ij})};\ i=1,2;\ j=c,p,l$$

From the comparison of estimators of both β_1 and β_2, the ML method was more accurate than the CD and PP methods. Except when the correlation between y and x_2 was low (0.2), the parameter estimates produced by the ML method were more accurate than those produced by the linear prediction method.

Hill and Ziemer (1983) studied four estimation methods for replacing missing regressor values under varying degrees of multicollinearity. The missing values estimation methods were the CLS, ZOR, MZOR and FOR. For each estimation method, the improvement in sampling variance given by (15) was calculated. Also, the bias of β introduced by using the data that included estimated missing values was calculated by using

$$\text{bias}(\hat{\beta}) = (X_c{}'X_c+X_mX_m{}')^{-1}X_m(X-X_m)'\hat{\beta}$$

Hill and Ziemer (1983) found that

1. In terms of bias and variance reduction, relative to the other estimation methods, the ZOR method gains significantly with increases in the degree of multicollinearity.

2. The estimation precision of the FOR method decreased rapidly as the degree of multicollinearity became acute.

3. Under a high degree of multicollinearity, the ZOR method performed better than the rest.

4. The gains from replacing missing values rather than discarding observations with missing values became greater when severe multicollinearity was present.

Azen et al. (1989) studied the CD, PP, and ML methods in the estimation of regression parameters and missing values. The ML method was implemented by the EM algorithm. They varied the degree of missing data (PM=5, and 25), the distribution of the independent variables (normal, mixture of normals, and lognormal), patterns of incomplete data (random, related, and censored), the intercorrelation coefficient (R=0.25 or 0.75), and the multiple correlation coefficient (R^2=0.5 or 0.9). The correlations between all pair of independent variables were the same. They assumed missing values occured only in the independent variables. The dependent variable was defined by the multiple linear regression of the form

$$y_i = \beta_0+\beta_1x_{i1}+\beta_2x_{i2}+\beta_3x_{i3}+e_i;\ i=1,\ldots,N$$

where, $\beta_j=1$, $j=0,1,2,3$ and e_i were iid $N(0,\sigma^2)$. Fifty data sets, each comprising a 100x3 matrix of N(0,1) variables were generated. Then, data with the desired distribution were obtained by transforming these data sets.

Azen et al. (1989) used two criteria, (23) and (24), for the evaluation of the estimation methods. The results were stratified by the distribution of the independent variables and the deletion pattern. Two-way analysis of variance for each criterion was conducted within each stratum.

For the estimation of the regression coefficients, Azen et al. (1989) found that

1. All three estimation methods performed well for moderate multiple correlation coefficients ($R^2=0.5$).

2. When $R^2=0.9$, the PP method was generally significantly inferior to the other two estimation methods at $\alpha=0.05$.

3. The CD method performed as well as the ML method, particulary when data were censored.

For the estimation of the mean vector and covariance matrix, Azen et al. (1989) found that

1. When $R^2=0.5$, all three methods performed well.

2. When $R^2=0.9$, the CD and PP methods were generally inferior ($P<0.05$) to the ML method.

Generally, the ML method performed better for the random and related patterns of incomplete data than for the censored data.

Summary

In summary, past research on the estimation methods has shown the following:

1. Among the indirect methods, the RG, PC and PP are comparable, and better than the MR and CD methods in estimating the variance-covariance matrix (Gleason & Staelin, 1975; Timm, 1970).

2. Lee and Chiu, (1990) in estimating polychoric correlations from incomplete data found that the direct methods of ML and PML gave better estimates than the indirect methods of CD, MR, RG, and PC. However, when the underlying data distribution was asymmetric the superiority of the direct methods was not highly pronounced.

3. When estimating regression parameters for both the linear and multiple regression, the ML procedure produces better estimates than any of the other methods. In the linear regression when the proportion missing x values or y values is less than 0.1, the CD estimates are comparable to the ML estimates. In the censored data the CD estimates are comparable to the ML estimates.

4. In estimating parameters of the simple linear regression model, the TSR estimates are comparable to the ML estimates.

What Does Literature Lack?

The literature shows the superiority of the direct ML method over the indirect methods (Lee & Chiu, 1990) for estimating various parameters of the regression model. For the estimation of the V-C matrix no one particular estimation method among the indirect methods consistently surpasses the rest (Gleason & Staelin, 1975; Timm, 1970). However, Timm (1970) and Gleason and Staelin (1975) found that as the average intercorrelation among the variables increased, the estimates produced by the PC and MR methods improved and were consistently better than the rest of the estimation methods. The CD method outperformed the remaining methods when the intercorrelations among the variables were low. Lee and Chiu (1990), studying estimation of polychoric correlations, found that the ML method had little advantage over the indirect methods when the data were asymmetric.

No single study for the comparison of the direct ML and the indirect methods in the estimation of the V-C matrix is available in the literature. However, Lee and Chiu (1990) have shown that the direct methods are generally superior to the indirect methods in estimating polychoric correlation matrix.

Among the investigations that focused on the regression model, only Azen et al. (1989) included nonnormal data. That study had at least three limitations. First, in each condition the correlation between each pair of independent variables was constant across pairs. Second, only the independent variables had missing data. Third, in the case of censored data the effect of left and right censoring was not investigated. Among the investigations that focused on the V-C or correlation matrix, only Lee and Chiu (1990) included nonnormal data and compared direct and indirect methods, but they studied estimation of polychoric correlation matrices.

CHAPTER 3
METHODOLOGY

In this chapter, the design of the study and the data generation procedure are described. Four indirect estimation methods, namely the CD, MR, PP, and RG, were compared to the direct ML method when some data were missing completely at random or censored. For the indirect methods, Σ and β were estimated by using ordinary least squares after the missing entries were estimated. For the direct method, estimation of β was by least squares after Σ was estimated.

The selection of the factors and levels in the design were based on the review of the relevant literature with the aim of making the experimental conditions used in the simulation similar to those found in educational research.

Design

The factors to be considered in the study were the degree of data skewness (SK), type of missing data (TM), sample size (N), number of independent variables (p), percent of missing data (PM), average intercorrelation of the variables (R), and residual variance (E). The levels of these factors are given in the following paragraphs.

Degree of data skewness (SK). Lee and Chiu (1990) showed that when the data distributions were symmetric, the direct methods gave better polychoric correlation estimates than did the indirect methods. However, the superiority of the direct methods over the indirect methods was negligible when the data were asymmetric.

Kendall and Stuart (1963, p.57) reported on age at time of marriage for over 300,000 Australians. They found that the data distribution had skewness and kurtosis of 2.0 and 8.3 respectively. Micceri (1989) investigated the distributional characteristics of 440 achievement and psychometric measures. In terms of the shape of the data distribution, Micceri found that 28% of the distributions were relatively symmetric, 41% were moderately asymmetric, and 31% were extremly asymmetric. Furthermore, 11% of the data distributions had skewness as extreme as 2.0.

In addition to generating regressor variables from the multivariate normal, regressor variables were generated from a multivariate lognormal distribution with the degree of skewness of 2.0. By calculating exp(y), with y normally distributed with mean zero and variance 0.304, lognormal data with skewness 2.0 and kurtosis 10.9 were constructed.

Sample size (N). Theoretically, as N increases, the stability of the estimates of Σ and of the regression parameters should increase. Hence, the CD and PP methods ought to yield progressively better estimates. The MR method is also sensitive to N since the efficiency of the mean of the observed data will increase with N. The effect of N on the RR and PP methods can be deduced from the fact that the estimation methods utilize the correlation matrix which is sensitive to an increase in N. As N increases, the likelihood function improves; this implies that the sample size has an influence on the direct methods as well. Timm (1970), and Gleason and Staelin (1975) noted that the performance of the estimation methods was strongly influenced by N. The levels of N were N=100 and 300.

Number of independent variables (p). The effect of the number of independent variables was also of interest. Data were generated using p=3, or 6 independent variables. This choice is consistent with the range of variables commonly studied in educational research (Algina & Oshima, 1990; Tang, 1990).

Percent missing data (PM). To simulate the incomplete data, some entries were deleted from the complete vector-valued data. Past research has shown that when PM>20 the estimation methods are likely to perform poorly (Gleason & Staelin, 1973; Timm, 1970). Azen et al. (1989) found that both the indirect (CD and PP) and direct (ML) estimation methods performed equally well when the percent of missing data in the independent variables was 5; the ML method had an edge over the other two estimation methods when PM=25, although the CD method was found to be safe and conservative. The levels of PM were 5% and 15%. In this study, data were missing from both the independent variables and dependent variable.

Type of missing data (TM). Missing values may arise in many ways. For example, the missing variables may be (a) forgotten and thus not measured, (b) measured and the values lost, or (c) unobservable through design considerations. Cases like (a) or (b) often give rise to data that are missing completely at random, while cases like (c) give rise to censored data. Three patterns of incomplete values were studied: missing completely at random, censored from left, and censored from right. For all three patterns, Nx(p+1)xPM/100 data points were deleted.

Average correlation among independent variables (R). Gleason and Staelin (1975) found that the performance of the estimation methods, particularly the indirect methods, was strongly influenced by the intercorrelation of the variables, R, as defined by equation (26). The RG and PP methods were reported to have performed well when the intercorrelation was high (R=0.6); while the MR did well when the intercorrelation was low ($0.1 \le R \le 0.3$) (Gleason &

Staelin, 1975). The influence of R on the direct methods is not very clear, but since the methods utilize the likelihood function, increase in R ought to improve the estimation of Σ and β. Timm (1970) studied data sets which had R between 0.2 and 0.8; while Gleason and Staelin (1975) varied R between 0.1 and 0.6.

This index, R, is highly related to the average correlation (Gleason & Staelin, 1975). In this study the average correlation ($\bar{r}$) was used in place of R. Although $\bar{r}$ ranges between 0 and 1, values of $\bar{r}>0.85$ are unrealistic and may cause multicollinearity of the variables. Among the consequences of multicollinearity are (a) regression coefficients tend to be poorly estimated and may have incorrect sign; (b) variances associated with the estimates of the regression coefficient may be greatly inflated; and (c) predicted values may be unrealistic. This factor $\bar{r}$, from this point onwards will be simply denoted as R. Thus, the levels of R were 0.2 and 0.7.

The correlation matrices **R** used in the study are presented in Table 1. The important features of these **R** matrices were that

1. The off-diagonal elements are not all equal.

2. Each matrix has a single dominant principal component, as shown by the eigenvalues.

3. All the eigenvalues are positive.

Table 1

Correlation Matrices

Number of variables: p=3

$$\boldsymbol{R}=\begin{bmatrix}1.00 & 0.11 & 0.33\\ 0.11 & 1.00 & 0.23\\ 0.33 & 0.23 & 1.00\end{bmatrix}$$

Eigenvalues: λ_i =1.46 0.90 0.64

Range of pairwise correlations: 0.11 - 0.33

Average correlation: R=0.22

Number of variables: p=3

$$\boldsymbol{R}=\begin{bmatrix}1.00 & 0.64 & 0.73\\ 0.64 & 1.00 & 0.82\\ 0.73 & 0.82 & 1.00\end{bmatrix}$$

Eigenvalues: λ_i = 2.46 0.37 0.16

Range of pairwise correlations: 0.64 - 0.82

Average correlation: R=0.73

Table 1--continued.

Number of variables: p=6

$$R=\begin{bmatrix} 1.00 & 0.11 & 0.33 & 0.23 & 0.11 & 0.33 \\ 0.11 & 1.00 & 0.23 & 0.11 & 0.33 & 0.23 \\ 0.33 & 0.23 & 1.00 & 0.33 & 0.23 & 0.11 \\ 0.23 & 0.11 & 0.33 & 1.00 & 0.11 & 0.33 \\ 0.11 & 0.33 & 0.23 & 0.11 & 1.00 & 0.23 \\ 0.33 & 0.23 & 0.11 & 0.33 & 0.23 & 1.00 \end{bmatrix}$$

Eigenvalues: λ_i =2.12 1.09 0.89 0.77 0.67 0.46

Range of pairwise correlation: 0.11 - 0.33

Average correlation: R=0.22

Number of variables: p=6

$$R=\begin{bmatrix} 1.00 & 0.64 & 0.82 & 0.73 & 0.64 & 0.82 \\ 0.64 & 1.00 & 0.73 & 0.64 & 0.82 & 0.73 \\ 0.82 & 0.73 & 1.00 & 0.82 & 0.73 & 0.64 \\ 0.73 & 0.64 & 0.82 & 1.00 & 0.64 & 0.82 \\ 0.64 & 0.82 & 0.73 & 0.64 & 1.00 & 0.73 \\ 0.82 & 0.73 & 0.64 & 0.82 & 0.73 & 1.00 \end{bmatrix}$$

Eigenvalues: λ_i = 4.65 0.53 0.36 0.27 0.18 0.01

Range of pairwise correlations: 0.64 - 0.82

Average correlation: R=0.73

Conditional variance (E). Conditional (residual) variance was set at two levels, E=6.0 or 12.0. These levels were chosen so that the coefficient of determination varied between 0.25 and 0.80.

Simulation Procedure

The initial factors included sample size, number of variables, degree of data skewness, average correlation among independent variables, and residual variance, all at 2 levels giving 32 conditions. The simulation was conducted as 32 separate runs, one for each condition, with 50 replications per condition; giving the full complete data. Then, for each condition 5% or 15% of the data were independently deleted at random (RM) or left censored (LC) or right censored (RC), giving data with missing values for 192 total conditions. For each condition the performance of the five estimation methods were evaluated.

A Multivariate Lognormal Distribution

Suppose **Y** is a p-variate normal distribution with means μ_i, variances σ_i^2, and correlations ϱ_{ij}, for i,j=1,...,p. Let $\mathbf{x}=(x_1,\ldots,x_p)'=[\exp(y_1),\ldots,\exp(y_p)]'$. Jones and Miller (1966) showed that **X** has a p-variate lognormal distribution with probability density function given by

$$f(\boldsymbol{X})=\frac{1}{|2\pi\Sigma|^{\frac{1}{2}}\prod_{i=1}^{p}\boldsymbol{X}_i}\exp\left(\frac{1}{2}(\log\boldsymbol{X}-\mu)'\Sigma^{-1}(\log\boldsymbol{X}-\mu)\right)$$

for $x_i>0$, $i=1,\ldots,p$ and $f(\mathbf{X})=0$ otherwise, and means, variances, and correlations defined as

$$\mu_i'=\exp\left(\mu_i+\frac{\sigma_i^2}{2}\right)$$

$$\sigma_i^{2'}=\exp\left(2\mu_i+2\sigma_i^2\right)-\exp\left(2\mu_i+\sigma_i^2\right)$$

$$\varrho_{ij}'=\frac{\exp(\varrho_{ij}\sigma_i\sigma_j)-1}{\left(\exp\sigma_i^2-1\right)^{\frac{1}{2}}\left(\exp\sigma_j^2-1\right)^{\frac{1}{2}}}$$

In the univariate case, skewness and kurtosis are defined as

$$\beta_1=\frac{E(x-\mu)^3}{(E(x-\mu)^2)^{\frac{3}{2}}}=\frac{\mu_3}{\sigma^3}$$

$$\beta_2=\frac{E(x-\mu)^4}{(E(x-\mu)^2)^2}=\frac{\mu_4}{\sigma^4}$$

respectively. For the lognormal distribution Mood et al. (1974) have given the rth moment about the origin as

$$\mu_r'=E(x^r)=\exp\left(r\mu+\frac{r^2\sigma^2}{2}\right)$$

Thus, the third central moment of a lognormal variable is

$$\mu_3'=E[(x-\mu')^3]=E(x^3)-3\mu'E(x^2)+3\mu'^2E(x)-\mu'^3$$

that is,

$$\mu_3'=\exp\left(3\mu+\frac{9}{2}\sigma^2\right)-3\exp\left(3\mu+\frac{5}{2}\sigma^2\right)+2\exp\left(3\mu+\frac{3}{2}\sigma^2\right)$$

and

$$\sigma^{3'}=(\exp(2\mu+2\sigma^2)-\exp(2\mu+\sigma^2))^{\frac{3}{2}}$$

The skewness of a lognormal variable is

$$\beta_1=\frac{\exp\left(3\mu+\frac{9}{2}\sigma^2\right)-3\exp\left(3\mu+\frac{5}{2}\sigma^2\right)+2\exp\left(3\mu+\frac{3}{2}\sigma^2\right)}{(\exp(2\mu+2\sigma^2)-\exp(2\mu+\sigma^2))^{\frac{3}{2}}}$$

The fourth central moment of a lognormal variable is

$$\mu_4'=E(x-\mu')^4=Ex^4-4\mu'Ex^3+6\mu^{2'}Ex^2-4\mu^{3'}Ex+\mu^{4'}$$

that is,

$$\mu_4'=\exp(4\mu+8\sigma^2)-4\exp(4\mu+5\sigma^2)+6\exp(4\mu+3\sigma^2)-3\exp(4\mu+2\sigma^2)$$

and

$$\sigma^{4'}=(\exp(2\mu+2\sigma^2)-\exp(2\mu+\sigma^2))^2$$

Thus, kurtosis of a lognormal variable is

$$\beta_2=\frac{\exp(4\mu+8\sigma^2)-4\exp(4\mu+5\sigma^2)+6\exp(4\mu+3\sigma^2)-3\exp(4\mu+2\sigma^2)}{(\exp(2\mu+2\sigma^2)-\exp(2\mu+\sigma^2))^2}$$

Following Johnson (1987), μ can be set to zero since it acts as a scale parameter for the lognormal distribution. With this specification β_1 and β_2 are determined by σ^2. Selecting $\sigma^2=0.304$ gives the $\beta_1=2.0$ and $\beta_2=10.9$.

Generation of the Data

Multivariate normal regressor variables were generated by using the following steps:

1. By p successive calls to a univariate normal generator,

$\mathbf{z} \sim N_p(\mathbf{0},\mathbf{I})$ was generated.

2. The px1 vector $\mathbf{x}$ was calculated by using $\mathbf{x}=\mathbf{Az}$ where $\mathbf{AA}'=\mathbf{\Sigma}$, $\mathbf{\Sigma}=\mathbf{DRD}$, $\mathbf{R}$ was one of the specified correlation matrices, Table 1, and $\mathbf{D}$ was a pxp diagonal matrix with $\mathbf{D}_{ii}=\sigma_i$ where $\sigma_i^2=0.304$. The distribution of $\mathbf{x}$ is $N_p(\mathbf{0},\mathbf{\Sigma})$.

The multivariate lognormal regressor variables were generated by using the following steps:

1. By p successive calls to a univariate N(0,1) generator, $\mathbf{z} \sim N_p(\mathbf{0},\mathbf{I})$ was generated.

2. The px1 vector $\mathbf{y}$ was calculated by using $\mathbf{y}=\mathbf{Az}$, where $\mathbf{AA}'=\mathbf{\Sigma}$. The distribution of $\mathbf{y}$ is $N_p(\mathbf{0},\mathbf{\Sigma})$.

3. The px1 vector $\mathbf{x}$ was calculated by using

$\mathbf{x}=(x_1,\ldots,x_p)'=[\exp(y_1),\ldots,\exp(y_p)]'$

The matrix $\mathbf{\Sigma}$ in step 2 was based on the following results. As noted earlier the mean and variance of a lognormal variable x=exp(y) are

$$\mu_i'=\exp\left(\mu_i+\frac{\sigma_i^2}{2}\right)$$

and

$$\sigma_i^{2\prime}=\exp\left(2\mu_i+2\sigma_i^2\right)-\exp\left(2\mu_i+\sigma_i^2\right)$$

where μ_i, σ_i^2 are the mean and variance for the normal variable y. Johnson, Ramberg, and Wang (1982) showed that

$$\varrho_{ij}=\frac{1}{\sigma_i\sigma_j}\ln\left(1+\frac{\varrho_{ij}'\sigma_i'\sigma_j'}{\mu_i'\mu_j'}\right)$$

where ρ_{ij}' is the correlation between $x_i=\exp(y_i)$ and $x_j=\exp(y_j)$ and ρ_{ij} is the correlation between y_i and y_j. For all conditions $\mu_i=0$ and $\sigma_i^2=0.304$. The parameter ρ_{ij}' was the ij th element of the correlation matrix (**R**) in Table 1. Using the relationship between ρ_{ij} and ρ_{ij}', the correlation matrix $\mathbf{R}^*$ was calculated. The matrix $\boldsymbol{\Sigma}$ was then calculated as $\boldsymbol{\Sigma}=\mathbf{DR}^*\mathbf{D}$ where $\mathbf{D}_{ii}=\sigma_i$.

4. Standardize the lognormal variable so that it has mean zero and variance $\sigma_i^2=0.304$.

For the regression analysis, $\mathbf{x}_{p+1}$ = **y** variable data were generated by pre-specifying the regression model

$$y_i = \beta_0+\beta_1x_1+\ldots+\beta_px_p+\epsilon_i \ ; \ i=1,\ldots,N$$

where, $\beta_0=\beta_1=\ldots=\beta_p=1$. The variable ϵ was iid normally distributed with mean zero and variance σ_ϵ^2 = 6.0 or 12.0.

Selection of Randomly Missing Data

To select PM values to delete from an Nx(p+1) complete data set, these steps were followed:

1. Generate a (0,1) random number.

2. Multiply the generated number by ((Nx(p+1))-1) and add 1 to generate a random number in (1,Nx(p+1)).

3. Round the number in step 2 to the nearest integer.

4. Delete the observation in the cell number given in step 3, counting along the rows of the complete data matrix.

5. Repeat steps 1 to 4 Nx(p+1)xPM/100 times to delete Nx(p+1)xPM missing values. In the event the same cell was identified more than once, steps 1 to 4 were repeated a sufficient number of times to delete Nx(p+1)xPM missing values. In addition a record with all variables missing was disallowed.

Selection of Censored Missing Data

Missing values were censored from either the left or the right of the data distribution. To identify data points for censoring, the following steps were used:

1. From the data set with randomly missing values, the number of missing values m_j in variable j, j=1,...,(p+1) was known.

2. From the complete data set for the jth variable (j=1,...,p+1), delete either the smallest m_j values resulting in left-censored data or the largest m_j values resulting in right-censored data.

Evaluation Techniques

The estimation methods were evaluated based on accuracy of estimation of (a) missing values, (b) V-C matrices, and (c) the multiple linear regression parameters.

Evaluation Based on the Estimation of Missing values

Gleason and Staelin (1975) defined the index of data reconstruction,

$$Q_\alpha = \sum_{i,j} \frac{\left(x_{ij}^{\alpha} - x_{ij}\right)^2}{\sigma_j^2 N(p+1)PM} \tag{30}$$

Estimation is better when Q_α is smaller.

Evaluation Based on the Accuracy of the V-C Estimates

The Gleason and Staelin (1975) modified Euclidean norm for the V-C matrix is defined as

$$D_\alpha = \left(\frac{\|\Sigma - \hat{\Sigma}_\alpha\|^2}{(p+1)^2} \right)^{\frac{1}{2}} \tag{31}$$

The estimation method with the smallest D_α is prefered. For the comparison of two estimation methods, Timm (1970) defined relative efficiency of method α to method β as

$$eff\left(\frac{\hat{\Sigma}_\alpha}{\hat{\Sigma}_\beta} \right) = \frac{\|\Sigma - \hat{\Sigma}_\alpha\|^2}{\|\Sigma - \hat{\Sigma}_\beta\|^2} \tag{32}$$

The estimation method α was preferable if the ratio was less than one, otherwise method β was the best.

Evaluation Based on Estimation of Regression Parameters

Two criteria for accuracy of estimated regression parameters were employed:

1. Let $\hat{\beta}_{j\alpha}$ denote the estimate of β_j calculated by applying method α and let $\hat{\beta}_j$ denote the estimate of β_j calculated by applying ordinary least squares to the data before deletion of the data. The criterion was

$$BSE(\alpha)=\frac{\sum_{j=0}^{p}(\beta_{j\alpha}-\beta_j)^2}{(p+1)} \tag{33}$$

2. The multiple linear regression model fit, defined as

$$s_\alpha^2=\frac{\sum_{i=1}^{N}\left(y_i-\beta_{0\alpha}-\sum_{j=1}^{p}\beta_{j\alpha}x_{ij}\right)^2}{(N-1)} \tag{34}$$

where $\hat{\beta}_{0\alpha}$ and $\hat{\beta}_{j\alpha}$, j=1,...,p, were the regression coefficients estimated by method α, x_{ij} and y_i were the true values of all variables before data deletion, and N was the sample size. Estimation is better when s^2_α is smaller. The quantity in equation (34) is an estimate of σ_ϵ^2. Thus, the ratio

$$RE=\frac{s_\alpha^2}{\hat{\sigma}_\epsilon^2} \tag{35}$$

of the estimated residual variance of the data that has been corrected for missing values to the residual variance of the original complete data was the criterion of interest. The closer to 1.0 the RE value is, the better the estimation.

CHAPTER 4
RESULTS AND DISCUSSION

In this chapter analyses of the data for the four criteria are presented. The criteria are (a) the mean squared deviations of regression coefficients for the data with estimated missing values from the regression coefficients for the original complete data (BSE); (b) the ratio ($s_\alpha^2/\hat{\sigma}_\epsilon^2$) of the estimated residual variance of the data with estimates for missing values to the residual variance of the original complete data (RE); (c) the modified euclidean norm (D); and (d) the index of data reconstruction (Q). For all the criteria except RE, smaller values indicate better estimation. For the RE criterion, values closer to 1.0 indicate better estimation. The analyses were based on the data presented in the Appendix, Table 30.

Analysis of variance was used to investigate the effect on the criteria of the following factors: sample size (N), number of dependent variables (p), degree of data skewness (SK), type of missing data (TM), percent of missing data (PM), average correlation among independent variables (R), residual variance (E), and estimation methods (M). The significance level quoted throughout this chapter is $\alpha=0.05$.

Development of the Working Model

There were five between-subjects factors (N, p, SK, R, E) and three within-subjects factors (TM, PM, M). All factors were considered fixed. Initial analyses were conducted to determine which effects to include in the analysis of variance model. A forward selection approach was used, with all main effects included first in the model, followed by all two-way interactions, all three-way interactions, and so on. The statistic used to compare the models was

$$F=\frac{\frac{FSS - RSS}{df_F - df_R}}{\frac{ESS}{df_E}} \tag{36}$$

where FSS and ESS are the model and error sum of squares, respectively, of the full model, df_F and df_E are the corresponding degrees of freedom, and RSS and df_R are the model sum of squares and degrees of freedom, respectively, of the reduced model. The quantity F is distributed as an F-statistic with (df_F-df_R) and df_E degrees of freedom.

Because there were between-subjects factors and within-subjects factors, it was important to determine first which between-subjects effects to enter into the model. Table 2 gives models having between-subjects effects only. There was no significant difference at $\alpha=0.05$ between the three-factor

Table 2

Magnitudes of R^2 for Between-Subjects Effects' Models

Variable	Highest-Order Terms	R^2
Q	Main Effects	0.038
	Two-Way Interactions	0.084
	Three-Way Interactions	0.133
	Four-Way Interactions	0.154
	Five-Way Interactions	0.158
D	Main Effects	0.143
	Two-Way Interactions	0.172
	Three-Way Interactions	0.173
	Four-Way Interactions	0.174
	Five-Way Interactions	0.174
RE	Main Effects	0.023
	Two-Way Interactions	0.059
	Three-Way Interactions	0.075
	Four-Way Interactions	0.082
	Five-Way Interactions	0.085
BSE	Main Effects	0.089
	Two-Way Interactions	0.149
	Three-Way Interactions	0.160
	Four-Way Interactions	0.165
	Five-Way Interactions	0.166

interaction model and the four-factor interaction model for any of the criteria. Results for criteria Q, RE and BSE suggested a model with up to three-way interactions, while the results for criterion D suggested a model with up to only two-way interactions. Thus, the two-way interaction model and three-way interaction model were the two possible candidates for the between-subjects factors.

To determine which within-subjects and mixed effects to enter, the forward selection approach was used again. Within-subjects and mixed effects were entered into purely between-subjects models. In Table 3, R_a^2 is the R^2 for the models with two-factor interactions as the most complicated between-subjects effects; R_b^2 is the R^2 for the models with three-factor interactions as the most complicated between-subjects effects. The R_a^2 values were lower than R_b^2 values, consequently models relating to the R_b^2 values were of interest. Using equation 36, the two-way and three-way interaction models were not significantly different at $\alpha=0.05$; while the main effects and the two-way interaction models were significantly different at $\alpha=0.5$ (Table 4). Thus, the model with within subjects main effects, and two-way interactions when the highest order of between subjects factors were the three-way interactions was selected. Table 5 shows the sources of variation for the selected model.

Using the selected model, and assuming that all factors were fixed, expected mean square components were computed

Table 3

Magnitudes of R^2 for Within-Subjects Main Effects, Two-Way Interaction, and Three-Way Interaction Models when the Highest-order Between-Subjects Factors Model are the Two-Way (R_a^2) or Three-Way (R_b^2) Interactions

Variable	Highest-Order Terms*	R_a^2	R_b^2
Q	Main Effects	0.618	0.734
	Two-Way Interactions	0.735	0.895
	Three-Way Interactions	0.746	0.909
D	Main Effects	0.940	0.945
	Two-Way Interactions	0.988	0.994
	Three-Way Interactions	0.991	0.998
RE	Main Effects	0.474	0.563
	Two-Way Interactions	0.686	0.867
	Three-Way Interactions	0.706	0.890
BSE	Main Effects	0.499	0.570
	Two-Way Interactions	0.752	0.886
	Three-Way Interactions	0.788	0.936

* Refers to highest-order terms for pure within-subjects effects.

Table 4

Model Comparisons

Models Compared	Criterion	df_F-df_R	df_E	F-test
3 vs 2	D	208	174	1.26
	RE	208	174	0.17
	BSE	208	174	0.03
	Q	104	102	0.16
2 vs ME	D	364	382	9.41*
	RE	364	382	2.41*
	BSE	364	382	2.91*
	Q	208	206	1.51*

* Means significant at α=0.05.
Note. Model 3 (2, ME) is the model with up to three-factor interactions (two-factor interactions, main effects).

Table 5

Skeleton Sources of Variation in the Analysis of Variance for the Selected Working Model

Source of Variation	
Between-Subjects Factors	
- Main Effects	(B1)
- Two-Way Interactions	(B2)
- Three-Way Interactions	(B3)
Error: Four-Way and Five-Way Interactions	(E_a)
Within-Subjects Factors	
- Main Effects	(W1)
- Two-Way Interactions	(W2)
- B1 x W1	
- B1 x W2	
- B2 x W1	
- B2 x W2	
- B3 x W1	
- B3 x W2	
Error: Remaining	(E_b)

for each effect. Since all the factors were assumed to be fixed, only the within-subjects error, $E_b = \sigma_\epsilon^2$, and the between-subjects error, $E_a = \sigma_\epsilon^2 + n_{TM} n_{PM} n_M \sigma_e^2$, were random where n_{TM}, n_{PM} and n_M are the number of levels for the factors TM, PM and M, respectively. The expected mean square for an effect A having a levels is

$$\sigma_{er}^2 + \frac{n\sum_{j=1}^{a} \alpha_j^2}{(a-1)}$$

where n is the product of the number of levels for the factors not included in the given effect, $\hat{\sigma}_{er}^2 = E_a$ if the effect involves only between-subjects factors, and $\hat{\sigma}_{er}^2 = E_b$ otherwise. Let

$$\phi_A^2 = \frac{\sum_{j=1}^{a} \alpha_j^2}{(a-1)}$$

The fixed component of an effect can be estimated as

$$\hat{\phi}_A^2 = \frac{MS_A - MS_E}{n}$$

where MS_E is the estimate of σ_{er}^2. Negative fixed components were set to zero. Myers (1979) recommended weighting the fixed component by df_A/n_A, where n_A is the number of levels of the effect, and df_A are the degrees of freedom of the effect. Thus,

$$\delta_A^2 = \frac{df_A \phi_A^2}{n_A}$$

The sum of the $\delta_A{}^2$s plus the estimates of $\sigma_\varepsilon{}^2$ and $\sigma_e{}^2$ give the total of the estimated components of the mean square, TMSE. The proportion $\delta_A{}^2$ to TMSE (W), expressed as a percentage, indicates the importance of the A effect relative to the rest. The proportions for each criterion will be reported in subsequent sections.

Many of the factors that would be expected to affect the criteria were included in the study. Consequently a large number of significant effects were expected. However, not all of these would necessarily be important. Initially, for each criterion an effect was considered important if (a) it was significant and (b) the W associated with the effect was large relative to the W associated with other effects that were significant for that criterion. The later part of the criterion for importance is analogous to the scree test in factor analysis. Inspection of W for all four criteria indicated that the definition of an important effect ranged from an effect with W=2.3% to an effect with W=12.9%. To compromise between interpreting a large number of unimportant effects and failing to interpret an important effect, the criterion used to select effects for interpretation was W>1.0%. This criterion has been used by Algina and Oshima (1994) and Coombs (1992).

Pairwise comparisons of means were conducted for effects that were significant and accounted for at least 1% of TMSE. The Tukey HSD test was used for pairwise comparisons. All the comparisons were done at $\alpha=0.05$ level of significance. Two means $\overline{Y}_i$ and $\overline{Y}_u$ were declared significantly different from each other if their absolute difference was greater than

$$W_t=q(\alpha\,;\,t,\nu)\hat{\sigma}_{\overline{y}}$$

where $\hat{\sigma}_{\overline{y}}$ was an estimate of the standard error of the mean based on ν degrees of freedom, t was the number of means, α was the significance level set at 0.05, and q(.) is the Studentized range critical value. The test controls the familywise error rate at α. There were two standard errors depending on whether the comparison involved one or two between-subjects treatments. Table 6 gives the formulae for estimating the various variances of the means and their degrees of freedom. Table 29 in the appendix gives the Tukey critical values at $\alpha=0.05$ for all pairwise comparisons.

Results for the Q Criterion

The Q criterion measures how well the missing values are reconstructed. It is computed by using equation (30). Data reconstruction is only used with the MR, RG, and EM methods. Reported in Table 7 are effects that (a) were

Table 6

Variances of the Means and their Degrees of Freedom

Source	Mean Variance	DF
Between-Subjects Means	$E_a(n_{TM}n_{PM}n_M)^{-1}$	E_a df
Within-Subjects Means	$E_b(n_N n_P n_{SK} n_R n_E)^{-1}$	E_b df
Within-Subjects Means for the same Between-Subjects Treatment	E_b	E_b df
Between-Subjects Means for the same Within-Subjects Treatment	$[(n_{TM}n_{PM}n_M-1)E_b+E_a](n_{TM}n_{PM}n_M)^{-1}$	Unknown

Table 7

Percent of TMSE for the Q Criterion

Effect	% Fixed Component
M	13.6
TM	12.9
TMxM	4.0
SKxTM	2.2
SKxRxTM	1.3
p	1.2
PM	1.1
pxTM	1.0

significant and (b) accounted for at least 1% of the TMSE. Estimation method and type of missing data had the dominant effects on Q score. Factors N and E had no apparent effect on Q score.

Effect of PM. Four factors had main effects that accounted for more than 1% of the TMSE. Of these only PM did not have a significant interaction with other factors. The marginal means for the two levels of PM were 2.16 for PM=5 and 2.86 for PM=15. Data reconstruction was better when the percent missing data was low. Other significant main effects were for M, TM, and p. However, since the TMxM, and pxM were significant, the effects of these factors were interpreted from the two-way tables.

The pxTM effect. A two-way table of means is presented in Table 8. For censored data, data reconstruction was significantly better when the number of independent variables was high, while for data missing at random, there were no significant differences in data reconstruction as p varied from 3 to 6. Data reconstruction was best when data were missing at random, and worst when data were right-censored regardless of the number of independent variables.

Table 8

Means of Q Scores as a Function of p and TM

		Type of Missing Data			
Factor	Level	RM	LC	RC	p Means
p	3	1.003 aA	3.648 bB	4.119 cB	2.923
	6	0.860 aA	1.854 bA	3.568 cA	2.094
TM Means		0.931	2.751	3.843	

Note. Within a row, means that do not have a common lower case letter are significantly different, and within a column, means that do not have a common upper case letter are significantly different at $\alpha=0.05$ using Tukey's HSD test.

The TMxM effect. The results are presented in Table 9. When data were missing at random, the EM and RG methods were significantly different with a lower Q mean score for the EM method; the MR method was not significantly different from either of the other two methods. When data were censored, the EM method was superior. The RG method outperformed the MR method only when data were right-censored. All estimation methods declined in performance when data were censored with poorest reconstruction when data were right-censored.

The SKxRxTM effect. A three-way table of means is presented in Table 10. For normally distributed data, data reconstruction was (a) superior when data were missing at random, and similar when data were left-censored and right-censored, regardless of the level of the average correlation, and (b) unaffected for all three types of missing data as the average correlation varied from 0.2 to 0.7. When data were lognormally distributed, the effect the estimation method depended on the average correlation. When the average correlation was 0.2 data reconstruction was superior when data were either left-censored or missing at random; when the average correlation was 0.7, data reconstruction was best with randomly missing data. Data reconstruction was (a) unaffected when data were missing at random as the average correlation varied from 0.2 to 0.7, (b) declined when data were left-censored as the average

Table 9

Means of Q Scores as a Function of TM and M

Factor Level		Estimation Method			TM Means
		EM	MR	RG	
TM	RM	0.350 aA	1.024 abA	1.420 bA	0.931
	LC	1.269 aB	3.784 bB	3.200 bB	2.751
	RC	2.426 aC	5.539 cC	3.565 bB	3.843
M Means		1.348	3.449	2.728	

Note. Within a row, means that do not have a common lower case letter are significantly different, and within a column, means that do not have a common upper case letter are significantly different at $\alpha=0.05$ using Tukey's HSD test.

Table 10

Means of Q Scores as a Function of SK, R, and TM.

Factors		Type of Missing data		
SK	R	RM	LC	RC
0	0.2	1.186 aA	3.088 bB	2.913 bA
0	0.7	0.707 aA	2.440 bB	2.676 bA
2	0.2	1.163 aA	1.723 aA	5.497 bC
2	0.7	0.669 aA	3.753 bC	4.287 bB

Note. Within a row, means that do not have a common lower case letter are significantly different. Within a column, in comparisons of levels of one factor controlling the other factors, means that do not have a common upper case letter are significantly different at $\alpha=0.05$.

correlation varied from 0.2 to 0.7, and (c) improved when data were right-censored as the average correlation varied from 0.2 to 0.7.

Summary. The results indicate that the EM and MR methods performed similarly when data were randomly missing, but when data were censored the EM method resulted in better reconstruction of missing data than did the MR and RG methods. The RG method gave better data reconstruction than the MR method when data were right-censored, otherwise the two estimation methods performed similarly. At both levels of p, reconstruction was best when data were missing at random and poorest when data were right-censored. Data reconstruction improved, though not significantly in the case of randomly missing data, with increase in the number independent variables. When data were normal, reconstruction was best when data were missing at random, and there was no significant difference between the two types of censoring at either levels of the average correlation. Reconstruction improved slightly, though not significantly, when the average correlation increased from 0.2 to 0.7 for all types of missing data. When the independent variables were lognormal, reconstruction was poorest when data were right-censored at both levels of the intercorrelation.

Results for the D Criterion

The D criterion measures how well the variance-covariance matrices are estimated. It is computed by using equation (31). Presented in Table 11 are effects that (a) were significant, and (b) accounted for at least 1% of the TMSE. Type of missing data and percent missing data had the dominant effects on D. The number of observations had no apparent effect on this criterion.

Effect of M. Five factors had main effects that accounted for more than 1% of the TMSE. Of these only M did not have a significant interaction with other factors. The marginal means for M were 0.466, 0.502, 0.598, 0.629 and 0.676 for EM, PP, CD, RG and MR respectively. Since Tukey's HSD critical value was 0.013, differences between all pairs of means were significant. The EM method was the best and MR was the poorest. The other significant main effects were TM, PM, E, and R, with each having significant interactions with other factors. Thus, the effects of these factors were interpreted from the two-way tables.

The SKxTM effect. A two-way table of means is presented in the top panel of Table 12. The estimation of the V-C matrices declined when data were censored for both normal and lognormal data. However, the estimation was better when data were left-censored than right-censored data only when

Table 11

Percent of TMSE for the D Criterion

Effect	% Fixed Component
TM	44.9
PM	19.1
E	10.2
M	4.0
ExTM	2.1
TMxPM	2.1
R	1.6
pxR	1.4
PMxE	1.2
SKxTM	1.1

Table 12

Means of D Scores as a Function of E and TM, TM and SK, and TM and PM.

Factor	Level	RM	LC	RC	Overall Means
SK	0	0.231 aB	0.784 bB	0.795 bA	0.603
	2	0.187 aA	0.623 bA	0.828 cB	0.546
E	6	0.163 aA	0.533 bA	0.651 cA	0.449
	12	0.254 aB	0.873 bB	0.971 cB	0.700
PM	5	0.117 aA	0.500 bA	0.593 cA	0.403
	15	0.301 aB	0.907 bB	1.030 cB	0.746
TM Means		0.209	0.703	0.811	

Type of Missing Data: RM, LC, RC

Note. Within a row, means that do not have a common lower case letter are significantly different, and within a column and panel, means that do not have a common upper case letter are significantly different at $\alpha=0.05$ using Tukey's HSD test.

data were lognormal. The estimation was better for lognormal data than normal data when data were missing at random or left-censored, but the opposite was the case when data were right-censored.

The TMxPM and ExTM effects. Two-way tables of means are presented in the lower two panels of Table 12. Estimation of the V-C matrices was best when (a) data were missing at random regardless of the level of E or PM, (b) percent missing data was low for all types of missing data, and (c) the conditional variance was small for all types of missing data.

The ExPM effect. A two-way table of means is presented in Table 13. Estimation of the V-C matrices was best when the percent of missing data was small, for both levels of the conditional variance. For both percent missing data, estimation was best when the conditional variance was smaller.

The pxR effect. A two-way table of means is presented in Table 14. At low average correlation, the estimation of the V-C matrices was best when the number of independent variables was high. At high average correlation, the estimation was unaffected as p increased from 3 to 6. When p was low, an increase in the average correlation did not

Table 13

Means of D Scores as a Function of PM and TM.

		Percent Missing Data		
Factor	Level	5	15	E Means
E	6	0.320 aA	0.578 bA	0.449
	12	0.486 aB	0.913 bB	0.700
PM Means		0.403	0.746	

Note. Within a row, means that do not have a common lower case letter are significantly different, and within a column, means that do not have a common upper case letter are significantly different at $\alpha=0.05$ using Tukey's HSD test.

Table 14

Means of D Scores as a Function of p and R.

		Number of Independent Variables		
Factor	Level	3	6	R Means
R	0.2	0.687 bA	0.490 aA	0.392
	0.7	0.695 aA	0.699 aB	0.465
p Means		0.691	0.594	

Note. Within a row, means that do not have a common lower case letter are significantly different, and within a column, means that do not have a common upper case letter are significantly different at $\alpha=0.05$ using Tukey's HSD test.

significantly affect the estimation, but when p was high, the estimation accuracy was significantly reduced with increase in the average correlation.

Summary. The results indicate that the EM method resulted in better estimation of the V-C matrices than did the other estimation methods. Estimation was better when the conditional variance, or percent missing data was small for all types of missing data. When the average correlation was low, estimation improved when the number of independent variables was high, but when the average correlation was high, the estimation was unaffected as p increased from 3 to 6. Estimation was best when data were missing at random for (a) normal and lognormal data, (b) small and large conditional variances, and (c) small and large percent missing data. Estimation was better for left-censored data than right-censored data regardless of the levels of E or PM, and for lognormal data. Of course, because of the symmetry of data distribution when data were normal, estimation was similar for left-censored data and right-censored data.

Results for the RE Criterion

The RE criterion measures how well the conditional variance is estimated. It is computed by using equation (35). The closer the RE scores are to 1, the better the estimation. Presented in Table 15 are effects that (a) were significant, and (b) accounted for at least 1% of the TMSE. Percent missing, type of missing data, and estimation method had dominant effects on RE, however none were outstanding.

Three factors had main effects that accounted for at least 1% of TMSE. All of these had significant interactions with other factors; consequently the effects of these factors were interpreted from tables of interaction effects.

The NxExTMxM effect. A four-way table of means is presented in Table 16. Tukey's HSD test was conducted for all factors but Table 16 reports only for the comparison of the five estimation methods. In general the differences between the methods at each combination of N, E, and TM were small, though a few significant differences were detected. The N effects, holding E, TM, and M constant were generally small. Similarly, the E, TM and M effects were small. Generally, for each combination of N and E, estimation accuracy of each method tended to decline, though not significantly, when data were censored, with right-censored data most affected.

The pxRxM effect. A three-way table of means is presented in Table 17. The results showed that (a) when p=3, all the estimation methods performed similarly regardless of the level of the average correlation, (b) when p=6 and

Table 15

Percent of TMSE for Criterion RE

Effect	% Fixed Component
PM	7.4
TM	4.8
M	3.1
SKxM	2.3
SKxTM	1.8
pxM	1.7
NxTMxM	1.5
NxExTMxM	1.5
pxRxM	1.4
TMxM	1.3
pxTM	1.2
PMxM	1.2
NxPM	1.1

Table 16

Means of RE Scores as a Function of N, E, TM, and M.

Factors			Estimation Methods				
N	E	TM	CD	EM	MR	PP	RG
100	6	RM	1.035 ab	0.972 a	0.976 a	1.058 b	1.048 b
100	6	LC	1.027 a	0.993 a	1.007 a	1.007 a	1.032 a
100	6	RC	1.117 b	1.046 a	1.034 a	1.043 a	1.216 c
100	12	RM	1.034 ab	0.967 a	0.969 a	1.047 b	1.026 ab
100	12	LC	1.038 a	0.994 a	1.003 a	1.009 a	1.007 a
100	12	RC	1.102 bc	1.025 a	1.027 a	1.035 ab	1.126 c
300	6	RM	1.008 a	0.992 a	0.996 a	1.026 a	1.033 a
300	6	LC	1.031 a	1.009 a	1.017 a	1.013 a	1.006 a
300	6	RC	1.050 a	1.014 a	1.020 a	1.271 c	1.099 b
300	12	RM	1.012 a	0.996 a	0.997 a	1.005 a	1.030 a
300	12	LC	1.043 a	1.020 a	1.024 a	1.020 a	1.012 a
300	12	RC	1.062 a	1.024 a	1.027 a	1.027 a	1.080 a
M Means			1.047	1.004	1.008	1.047	1.060

Note. Within a row, means that do not show a common letter are significantly different at $\alpha=0.05$ using Tukey's HSD test. Within a column, when N and E are fixed the critical value for comparing TM means is 0.146, when N and TM are fixed the critical value for comparing E means is 0.122, and when E and TM are fixed the critical value for comparing N means is 0.122.

Table 17

Means of RE Scores as a Function of p, R, and M.

Factors		Estimation Methods				
p	R	CD	EM	MR	PP	RG
3	0.2	1.043 aA	1.014 aA	1.017 aA	1.016 aA	1.018 aA
3	0.7	1.031 aA	1.003 aA	1.004 aA	1.025 aA	1.021 aA
6	0.2	1.072 bB	1.003 aA	1.008 aA	1.011 aA	1.033 aA
6	0.7	1.041 bA	0.998 aA	1.003 abA	1.135 cB	1.166 cB
M Means		1.047	1.004	1.008	1.047	1.060

Note. Within a row, means that do not have a common letter are significantly different. Within a column, in comparisons of levels of one factor controlling the other factors, means that do not have a common upper case letter are significantly different at $\alpha=0.05$.

R=0.2, all the estimation methods performed similarly except for the CD method which performed more poorly than the rest, and (c) when p=6 and R=0.7, the EM and MR methods performed similarly and were the best, while the PP and RG methods were the poorest. The p and R factors affected the estimation accuracy only for the CD, PP and RG methods. With p=6, accuracy of the PP and RG methods declined while that of the CD method improved, as the correlation increased.

The SKxTM, and pxTM effects. Two-way tables for interpreting the SKxTM and pxTM effects are reported in Table 18. The estimation of the conditional variance was similar for missing at random data and left-censored data regardless of the degree of skewness or the number of independent variables. When data were left-censored, estimation was better for lognormal data than for normal data. The opposite was the case when data were right-censored. As p increased from 3 to 6, estimation from the randomly missing data and the right-censored data became poorer while that from the left-censored data was unaffected.

The NxPM effect. A two-way table of means is presented in Table 19. The estimation of the conditional variance was significantly poorer as the percent missing data increased from 5% to 15% regardless of the sample size. When the

Table 18

Means of RE Scores as a Function of SK and TM, and p and TM.

		Type of Missing Data			Overall
Factor	Level	RM	LC	RC	Means
SK	0	1.012 aA	1.028 abB	1.044 bA	1.028
	2	1.011 aA	1.004 aA	1.101 bB	1.039
p	3	0.999 aA	1.018 aA	1.041 bA	1.353
	6	1.024 aB	1.013 aA	1.104 bB	1.047
TM Means		1.011	1.016	1.072	

Note. Within a row, means that do not have a common letter are significantly different, and within a column and panel, means that do not have a common upper case letter are significantly different at $\alpha=0.05$ using Tukey's HSD test.

Table 19

Means of RE Scores as a Function of N and PM

		Percent Missing Data		
Factor	Level	5	15	N Means
N	100	0.986 aA	1.082 bB	1.034
	300	1.011 aA	1.053 bA	1.032
PM Means		0.999	1.067	

Note. Within a row, means that do not have a common letter are significantly different, and within a column, means that do not have a common upper case letter are significantly different at $\alpha=0.05$ using Tukey's HSD test.

percent missing data was low, the estimation slightly improved but not significant at $\alpha=0.05$, as sample size increased. When the percent missing data was high, the estimation significantly improved as sample size increased.

The SKxM, and PMxM effects. Two-way tables of means are presented in Table 20. From the SKxM table of means, the results showed that (a) for normally distributed data, the EM, MR and PP methods performed similarly and were the best, (b) for lognormally distributed data, the EM and MR performed similarly and were the best, and (c) only the PP method was affected by the skewness of the data with the performance declining as skewness increased from 0 to 2.

From the PMxM table of means, the results showed that (a) when the percent missing data was low, all the estimation methods performed similarly, (b) when the percent missing data was high, the EM and MR performed similarly and were the best, while the RG method was the worst, and (c) the performance of all the estimation methods declined as percent missing data increased from 5% to 15%.

Summary. The results indicate that (a) when data were normally distributed, the EM, MR and PP methods performed similarly and were the best, and when data were lognormally distributed, the EM and MR methods were the best, (b) when percent missing data was low, all the estimation methods

Table 20

Means of RE Scores as a Function of SK and M, and PM and M

Factor Level	Estimation Methods CD	EM	MR	PP	RG	Overall Means
SK 0	1.041 bA	1.000 aA	1.010 aA	1.020 aA	1.067 bA	1.028
2	1.052 bA	1.009 aA	1.006 aA	1.073 bB	1.052 bA	1.038
PM 5	0.996 aA	0.986 aA	0.989 aA	1.014 aA	1.009 aA	0.999
15	1.098 bcB	1.023 aB	1.027 aB	1.079 bB	1.111 cB	1.068
M Means	1.047	1.004	1.008	1.047	1.060	

Note. Within a row, means that do not have a common letter are significantly different, and within a column and panel, means that do not have a common upper case letter are significantly different at $\alpha=0.05$ using Tukey's HSD test.

performed similarly, but when percent missing data was high, the EM and MR performed similarly and were the best, (c) the performance of all the estimation methods declined as the percent missing data increased from 5% to 15%, and (d) increase in skewness negatively affected the PP method only. The estimation improved as the sample size increased from 100 to 300 when the percent missing data was low, while the opposite was the case when the percent missing data was high. The results showed that the performance of the PP and RG methods were negatively affected by increase in both p and R, while the performance of the EM, MR and CD methods were unaffected.

Results for the BSE Criterion

The BSE criterion measures the mean squared deviations of regression coefficients for the data with estimated missing values from the regression coefficients for the original complete data. It is computed by using equation (33). Smaller values indicate better estimation method. Reported in Table 21 are effects that (a) were significant and (b) accounted for at least 1% of the TMSE. The significant main effects were for R, M, p, and PM. However, since these factors had significant interactions with other factors, the effects of these factors will be interpreted from tables of interaction effects. No effects involving N or E satisfying the above criteria were found.

Table 21

Percent of TMSE for the BSE Criterion

Effect	% Fixed Component
pxPMxM	8.9
R	4.7
pxR	4.3
RxM	3.9
pxM	3.7
M	3.5
pxRxM	2.8
p	2.7
TMxM	2.3
PM	2.0
RxPMxM	1.8
PMxM	1.7
RxTMxM	1.7
SKxTMxM	1.1

The pxPMxM effect. A three-way table of means is presented in Table 22. When p=3, all the estimation methods performed similarly regardless of the level of PM, except the CD method which was inferior to the rest when percent missing data was high. When p=6, the EM, MR and CD methods performed similarly and were the best regardless of the percent missing data. Fixing the percent missing data, the results showed that as the number of independent variables increased from 3 to 6, the performance of (a) the EM, MR and CD was unaffected, and (b) the PP and RG methods declined.

When the number of independent variables are fixed, the results showed that as the percent missing data increased, the performance of (a) the CD method declined regardless of the number of independent variables, (b) the PP and RG methods declined only when p=3 and p=6 respectively, and (c) the EM and MR methods declined, though the difference was not statistically significant.

The pxRxM effect. A three-way table of means is presented in Table 23. When p=6 and R=0.7, the EM, MR, and CD methods performed similarly and better than the PP and RG methods. For other combinations of p and R, all methods performed similarly.

When the intercorrelation increased from 0.2 to 0.7, the performance of all the estimation methods was (a) unaffected when p=3, and (b) reduced when p=6. When the

Table 22

Means of BSE Scores as a Function of p, PM, and M

Factors		Estimation Methods				
p	PM	CD	EM	MR	PP	RG
3	5	0.025 aA	0.013 aA	0.019 aA	0.016 aA	0.021 aA
3	15	0.150 bB	0.055 aA	0.088 aAB	0.100 aB	0.078 aA
6	5	0.067 aA	0.049 aA	0.052 aAB	0.228 bC	0.166 bB
6	15	0.131 abB	0.082 aA	0.092 aB	0.201 bC	0.665 cC
M Means		0.093	0.050	0.063	0.136	0.227

Note. Within a row, means that do not have a common letter are significantly different. Within a column, in comparisons of levels of one factor controlling the other factors, means that do not have a common upper case letter are significantly different at $\alpha=0.05$.

Table 23

Means of BSE Scores as a Function of p, R, and M

Factors		Estimation Methods				
p	R	CD	EM	MR	PP	RG
3	0.2	0.089 aAB	0.039 aA	0.062 aAB	0.045 aA	0.031 aA
3	0.7	0.086 aA	0.029 aA	0.045 aA	0.071 aA	0.068 aA
6	0.2	0.040 aA	0.013 aA	0.022 aA	0.015 aA	0.035 aA
6	0.7	0.159 aB	0.118 aB	0.123 aB	0.413 bB	0.796 cB

Note. Within a row, means that do not have a common letter are significantly different. Within a column, in comparisons of levels of one factor controlling the other factors,means that do not have a common upper case letter are significantly different at $\alpha=0.05$.

number of independent variables increased from 3 to 6, the performance of all the estimatiom methods was (a) unaffected when R=0.2, and (b) reduced when R=0.7.

The RxPMxM effect. A three-way table of means is reported in Table 24. When R=0.2, all the estimation methods performed similarly regardless of the level of PM. When R=0.7, the EM, MR and CD methods performed similarly and were the best regardless of the level of PM. As the percent missing data increased from 5% to 15%, only the CD method declined in performance. The EM, MR and PP methods were unaffected regardless of the average correlation. Performance of the RG method declined only when the average correlation was high. Similarly, with percent missing data fixed, as the average correlation increased, the performance of (a) the PP and RG methods declined regardless of the percent missing data, (b) the CD method declined only when the percent missing data was high, and (c) the EM and MR methods were unaffected.

The RxTMxM effect. A three-way table of means is presented in Table 25. When R=0.2, all the estimation methods performed similarly regardless of the type of missing data. When R=0.7, similar results were observed except that the RG method was inferior to the rest, and the PP method performed poorly when data were missing at random.

Table 24

Means of BSE Scores as a Function of R, PM, and M.

Factors		Estimation Methods				
R	PM	CD	EM	MR	PP	RG
0.2	5	0.020 aA	0.009 aA	0.014 aA	0.011 aA	0.013 aA
0.2	15	0.108 aB	0.042 aAB	0.070 aAB	0.049 aA	0.053 aA
0.7	5	0.072 aAB	0.053 aAB	0.057 aAB	0.233 bB	0.174 bB
0.7	15	0.173 aC	0.095 aB	0.110 aB	0.252 bB	0.691 cC

Note. Within a row, means that do not have a common letter are significantly different. Within a column, in comparisons of levels of one factor controlling the other factors, means that do not have a common upper case letter are significantly different at $\alpha=0.05$.

Table 25

Means of BSE Scores as a Function of R, TM, and M.

Factors		Estimation Methods				
R	TM	CD	EM	MR	PP	RG
0.2	RM	0.005 a	0.002 a	0.007 a	0.002 a	0.026 a
0.2	LC	0.090 a	0.043 a	0.065 a	0.047 a	0.011 a
0.2	RC	0.098 a	0.032 a	0.054 a	0.040 a	0.062 a
0.7	RM	0.080 a	0.073 a	0.040 a	0.552 b	0.494 b
0.7	LC	0.099 a	0.055 a	0.120 a	0.086 a	0.278 b
0.7	RC	0.188 a	0.093 a	0.091 a	0.089 a	0.525 b

Note. Within a row, means that do not show a common letter are significantly different. Within a column, in comparisons of levels of one factor controlling the other factors, means that di not have a common upper case letter are significantly different at $\alpha=0.05$.

The PP method, especially when data were missing at random, and the RG method were heavily affected by an increase in average correlation regardless of the type of missing data.

The SKxTMxM effect. A three-way table of means is presented in Table 26. When the data were normal, all the estimation methods except the RG method performed similarly regardless of the type of missing data. When the data were lognormal, with the following exceptions the estimation methods performed similarly regardless of the type of missing data: (a) when data were randomly missing, then the performance of the PP and RG methods was significantly poorer, and (b) when data were right-censored, then the performance of the RG method was significantly poorer. The performance of all the estimation methods, except the PP and RG methods, declined though not significantly, when data were censored.

Summary. The results of the BSE criterion showed that as the percent missing data increased from 5% to 15%, the performance of (a) the CD method declined regardless of the level of p, (b) the PP method declined only when p=3, (c) the RG method declined only when p=6, and (d) the EM and MR methods slightly declined, though not significantly at $\alpha=0.05$, regardless of the level of p. Furthermore, as the number of independent variables increased from 3 to 6, the

Table 26

Means of BSE Scores as a Function of SK, TM, and M

Factors		Estimation Methods				
SK	TM	CD	EM	MR	PP	RG
0	RM	0.046 a	0.041 a	0.017 a	0.083 a	0.219 b
0	LC	0.098 a	0.037 a	0.067 a	0.057 a	0.234 b
0	RC	0.112 a	0.053 a	0.067 a	0.054 a	0.365 b
2	RM	0.040 a	0.033 a	0.030 a	0.472 c	0.300 b
2	LC	0.090 a	0.062 a	0.117 a	0.077 a	0.056 a
2	RC	0.175 ab	0.072 a	0.079 a	0.075 a	0.221 b

Note. Within a row, means that do not have a common letter are significantly different at $\alpha=0.05$ using Tukey's HSD test. Within a column, when SK is fixed the critical value for comparing TM means is 0.103, and when TM is fixed the critical value for comparing SK means is 0.086.

performance of (a) the EM, MR and CD was unaffected, and (b) the PP and RG methods declined for all levels of PM.

As the number of independent variables increased from 3 to 6, the performance of all the estimation methods was (a) unaffected when the average correlation was low, and (b) declined when the average correlation was high.

Summary with Respect to the Estimation Methods

For estimating V-C matrices, there were no interactions with estimation methods. However, comparisons among the estimation methods showed that the EM method was the most accurate. For the other criteria--reconstruction of missing values (Q), estimation of the conditional variance (RE), and estimation of the regression coefficients (BSE)--there were significant interactions with estimation methods. For each criterion and each significant interaction with estimation methods, the most accurate estimation methods are displayed in Table 27.

Table 27

The Best Estimation Methods as Other Factors are Varied

Factors and Levels		Criteria: Q	RE	BSE
TM	RM	EM MR		
	LC	EM		
	RC	EM		
SK	0		EM MR PP	
	2		EM MR	
PM	5		CD EM MR PP RG	
	15		EM MR	
p	R			
3	0.2		CD EM MR PP RG	CD EM MR PP RG
3	0.7		CD EM MR PP RG	CD EM MR PP RG
6	0.2		EM MR PP RG	CD EM MR PP RG
6	0.7		EM MR	CD EM MR

Table 27 -- Continued.

Factors and Levels		Criteria
p	PM	BSE
3	5	CD EM MR PP RG
3	15	EM MR PP RG
6	5	CD EM MR
6	15	CD EM MR
R	PM	
0.2	5	CD EM MR PP RG
0.2	15	CD EM MR PP RG
0.7	5	CD EM MR
0.7	15	CD EM MR
R	TM	
0.2	RM	CD EM MR PP RG
0.2	LC	CD EM MR PP RG
0.2	RC	CD EM MR PP RG
0.7	RM	CD EM MR
0.7	LC	CD EM MR PP
0.7	RC	CD EM MR PP

Table 27 -- Continued.

Factors and Levels		Criteria
		BSE
SK	TM	
0	RM	CD EM MR PP
0	LC	CD EM MR PP
0	RC	CD EM MR PP
2	RM	CD EM MR
2	LC	CD EM MR PP RG
2	RC	CD EM MR PP

Table 27 -- Continued.

Factors and Levels			Criteria
			RE
N	E	TM	
100	6	RM	CD EM MR
100	6	LC	CD EM MR PP RG
100	6	RC	EM MR PP
100	12	RM	CD EM MR RG
100	12	LC	CD EM MR PP RG
100	12	RC	EM MR PP
300	6	RM	CD EM MR PP RG
300	6	LC	CD EM MR PP RG
300	6	RC	CD EM MR
300	12	RM	CD EM MR PP RG
300	12	LC	CD EM MR PP RG
300	12	RC	CD EM MR PP RG

CHAPTER 5
CONCLUSIONS

One hundred ninety-two simulated conditions were used to compare the effects on four measures of performance of five estimation methods. Although three methods, namely CD, MR and PP, are ad hoc in character, they are commonly used in missing value problems. On the other hand, the EM algorithm and the RG method are fortified with statistical arguement and derivation. The use of the EM method is often avoided because of its complexity and demanding computing skills.

The four measures of performance were (a) the mean sum of squares of deviations of regression coefficients of the data that has been corrected for missing values from the regression coefficients of the original complete data, BSE; (b) the ratio $(s^2/\hat{\sigma}_\epsilon^2)$ of the estimated residual variance of the data that has been corrected for missing values to the residual variance of the original complete data, RE; (c) the modified euclidean norm, D; and (d) the index of data reconstruction, Q. Missing data are not reconstructed when the CD and PP methods are used, consequently the CD and PP methods were not involved when the Q criterion results were considered.

The results obtained may be applied to experiments which have experimental conditions similar to the studied 192 conditions. Consequently, the generalizability of the results is limited by the range of values for the degree of data skewness (SK), average intercorrelation of variables (R), percent of missing entries (PM), type of missing data (TM), residual variance (E), sample size (N), and number of independent variables (P). Within these limitations, the following findings of the present study and comparisons to previous literature can be made.

Based on the Estimation of Missing Values

Past research has shown that

1. The MR method was better than the RG method when the average correlation of the independent variables was less than 0.2, otherwise the RG method was superior (Gleason & Staelin, 1975).

2. When the percent missing data was small (PM=5), the EM, CD and PP methods performed similarly, except when correlational structure was high ($R=0.75$, $R^2=0.9$), then the PP method was no longer competitive (Azen et al., 1989).

3. When percent missing data was large (PM=25), the EM method was better than the CD and PP methods (Azen et al., 1989).

4. Data reconstruction was poorer when data were lognormal or censored (Azen et al., 1989).

The results of the present study have shown that:

1. Data reconstruction was better when the percent missing data was low.

2. Reconstruction was better when data were missing at random than censored data regardless of the degree of skewness, average correlation, or number of independent variables.

3. The EM method gave the best data reconstruction for all types of missing data, and the MR method performed similarly as (a) the EM method only when data were missing at random, and (b) the RG method except when data were right-censored then the MR method was the poorest.

Thus, the present study (a) verifies the superiority of the EM method, and (b) shows that among the ad hoc methods the MR method is as well as the EM method when data are randomly missing, (c) shows that when data are censored, the MR is no longer competitive, and (d) shows that the performance of all the estimation methods declines when data are censored.

Based on the Estimation of Regression Coefficients

This has been the most commonly used criterion for assessing the performance of the estimation methods. Past research has shown that

1. Based on the analysis of pivotal (equation 29), the EM method performed better than the CD method especially when the percent missing data was large (Little, 1979).

2. Based on bias of the regression coefficients, $(\hat{\beta}-\beta)$, the CD method performed just as well as the MR and RG methods when the average correlation of the variables was low (R=0.2). When the average correlation was high (R=0.9), the MR method was superior to the CD and RG methods (Hill & Ziemer, 1983).

3. The CD, EM and PP methods performed similarly regardless of the correlational structure when the percent missing data was low (Azen et al., 1989). When the percent missing data was high, the EM method performed better than the rest.

The results of the present study have shown that

1. When the number of independent variables was low, all the estimation methods performed similarly regardless of the percent missing data or the average correlation. When the number of independent variables was high, (a) all the estimation methods performed similarly when the average correlation was low, and (b) the CD, EM and MR methods performed similarly and were the best when the average correlation was high, or regardless of the percent missing data.

2. The performance of all the estimation methods declined as the percent missing data increased from 5% to

15% regardless of (a) the number of independent variables, with the trend being nonsignificant for the EM and MR methods, and for the PP method when p=6, or (b) the average correlation, with the trend being significant only for the CD method, and for the RG method when the average correlation was high.

3. The performance of all the estimation methods declined as the average correlation increased from 0.2 to 0.7 (a) regardless of the number of independent variables, however when the number of independent variables was low, this trend was not significantly for the PP and RG methods, and (b) for all types of missing data, though not significantly except for the RG method and for the PP method when data were missing at random.

Thus, the present study (a) verifies the superiority of the EM method, (b) shows that among the ad hoc methods, the CD and MR methods are as good as the EM method in most situations, and (c) shows that the estimation is poorer when data are censored than when data are missing at random.

Based on the Estimation of the Conditional Variance

Very few studies have used this criterion for assessing the performance of the estimation methods. Beale and Little (1975) found that the EM and RG method consistently gave better estimate of the residual than the CD method, and that the EM method improved on the RG method.

The results of the present study have shown that

1. The EM, MR and PP methods performed similarly and were the best when data were normally distributed. When data were lognormally distributed, the EM and MR outperformed the rest. The impact of skewness on the estimation methods was negligible except for the PP method whose performance declined with increase in skewness.

2. When the percent missing data was low, all the estimation methods performed similarly. When the percent missing data was high, the EM and MR methods were superior to the rest. However, the performance of all the methods declined when the percent missing data increased from 5% to 15%.

3. The estimation improved with increase in sample size, particularly when the percent missing data was high.

4. All the estimation methods performed similarly regardless of the level combination of p and R except when p=6 and R=0.7 the PP and RG methods were inferior to the rest.

Based on the Estimation of the V-C matrix

This criterion was used in the early work of Timm (1970), and Gleason and Staelin (1975). From Timm (1970), for the estimation of the V-C matrices, the results favored the RG method over the CD and MR method when data were randomly missing. No clear pattern by the estimation methods was noted as the intercorrelation, percent missing data, number of variables, or sample size varied.

From Gleason and Staelin (1975), for the estimation of the correlation matrices, the results showed that

1. When the average correlation was low (R<0.2) the CD method performed better than the RG, MR and PP methods.

2. When R>0.2 the RG method performed better than the rest.

The results of the present study have shown that:

1. The EM method was the best regardless of the presence of the other factors.

2. The estimation was better when data were missing at random than when data were censored with right-censored data being the poorest.

3. The estimation declined as the conditional variance, percent missing data, or average correlation increased.

4. The estimation improved when the number of independent variables increased only when the average correlation was low.

The results of the present study seem to point out one other important feature of the criteria, that the criterion based on the estimation of the regression coefficients did not discriminate the estimation methods as much as the other criteria did (Table 27). Probably this was due to that for the BSE criterion the estimation methods were interpreted from the three-way interactions only.

Suggestions to Future Research

Based on the Q, BSE, or RE criteria, estimation accuracy declined substantially when data were censored; the CD, MR and PP methods were most affected. However, when data were missing at random, the CD, MR and PP methods were close contenders to the EM method. This suggests that future research should investigate the degree of censoring the estimation methods will tolerate and also investigate the impact of other conditions in which data are not missing at random.

Second, the results have shown that the effect of skewness is minimal except when estimation of missing values is the criterion, while the effect of R or PM is very strong. Thus, it seems that further research on SK may not be as critical as studying a wider variety of levels of R or PM. In this study the choice of the structure of the variables correlation matrix was arbitrary as long as R was either 0.2 or 0.7. Further research is needed to investigate the impact of varying structures of the correlation matrix. Though situations with R>0.7 may be unrealistic due to multicollinearity, it is important to investigate the level at which multicollinearity becomes a limiting factor in the ability of the estimation methods.

Third, sizes N=100 and 300 were included in the study. The effect of the sample size on the estimation methods was not strong. It may be that for p=3 and 6, and PM=5 and 15, N=100 was sufficiently high. Further research might include smaller N values for these values of p and PM and larger N values for larger values of p and PM.

The superiority of the EM method over all the indirect methods, suggests that future research should focus on estimation methods that utilize the iterative procedures. The advantages of estimating missing values through iteration have been demonstrated by the performance of the EM method. Other possible iterative type of procedures include (a) Dear's principal component method, (b) the general iterative principal component method, and (c) the singular value decomposition method.

APPENDIX

Table 28 shows abbreviations used. Table 29 presents the Tukey calculated critical values at $\alpha=0.05$. Table 30 presents the D, RE, BSE, and Q criteria scores.

Table 28

Abbreviations

Abbreviation	Meaning
CD	complete data only
CLS	classical least squares
EM	expectation-maximization
FOR	first-order regression
MAR	missing at random
MCAR	missing completely at random
ML	maximum likelihood
MR	mean replacement
MZOR	modified zero-order regression
PC	principal components
PP	complete pair variable only
RG	regression replacement
TMSE	Total of estimated mean square components
TSR	two-stage regression
V-C	variance-covariance
ZOR	zero-order regression
LS	least squares
PML	pairwise maximum likelihood
MPQ	mean pivots of the regression parameters
MSPQ	mean sum of squares of the pivots

Table 29

Tukey HSD Critical Values at $\alpha=0.05$

Effect		Q	D	RE	BSE
SK, R, E, N, p		0.761	0.022	0.020	0.039
SKxR,...,Nxp		1.076	0.030	0.028	0.055
TM		0.394	0.009	0.013	0.033
PM		0.269	0.006	0.009	0.022
M		0.394	0.013	0.020	0.049
TMxM	- simple effect of M	0.683	0.023	0.035	0.085
	- simple effect of TM	0.683	0.019	0.030	0.073
TMxPM	- simple effect of PM	0.467	0.010	0.016	0.039
	- simple effect of TM	0.558	0.012	0.019	0.046
PMxM	- simple effect of PM	0.467	0.013	0.020	0.050
	- simple effect of M	0.558	0.019	0.028	0.070
AxTM	- simple effect of TM	0.558	0.012	0.019	0.046
	- simple effect of A	0.467	0.010	0.016	0.039
AxPM	- simple effect of A	0.381	0.008	0.013	0.032
	- simple effect of PM	0.381	0.008	0.013	0.032
AxM	- simple effect of A	0.467	0.013	0.020	0.050
	- simple effect of M	0.558	0.019	0.028	0.070
AxBxTM	- simple effect of A or B	0.660	0.015	0.022	0.055
	- simple effect of TM	0.789	0.017	0.027	0.065

Table 29--continued.

Effect		Q	D	RE	BSE
AxBxM	- simple effect of A or B	0.660	0.019	0.029	0.071
	- simple effect of M	0.920	0.026	0.040	0.098
AxTMxM	- simple effect of A	0.808	0.023	0.035	0.086
	- simple effect of TM	0.966	0.028	0.042	0.103
	- simple effect of M	1.126	0.032	0.049	0.120
AxPMxM	- simple effect of A	0.660	0.019	0.029	0.071
	- simple effect of PM	0.660	0.019	0.029	0.071
	- simple effect of M	0.920	0.026	0.040	0.098
AxBxTMxM	- simple effect of A or B	1.143	0.033	0.050	0.122
	- simple effect of TM	1.366	0.039	0.060	0.146
	- simple effect of M	1.592	0.045	0.069	0.170

Note. A, B = SK or R or E or N or p

Table 30

The BSE, D, Q, and RE Criteria Scores

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	0	LC	0.2	6	5	100	3	0.025	0.501	0.000	1.016
CD	0	LC	0.2	6	5	100	6	0.005	0.441	0.000	0.976
CD	0	LC	0.2	6	5	300	3	0.036	0.350	0.000	0.996
CD	0	LC	0.2	6	5	300	6	0.014	0.302	0.000	1.030
CD	0	LC	0.2	6	15	100	3	0.171	0.922	0.000	1.163
CD	0	LC	0.2	6	15	100	6	0.012	0.655	0.000	1.103
CD	0	LC	0.2	6	15	300	3	0.138	0.599	0.000	1.064
CD	0	LC	0.2	6	15	300	6	0.041	0.474	0.000	1.102
CD	0	LC	0.2	12	5	100	3	0.042	0.678	0.000	1.027
CD	0	LC	0.2	12	5	100	6	0.018	0.562	0.000	1.002
CD	0	LC	0.2	12	5	300	3	0.061	0.661	0.000	1.016
CD	0	LC	0.2	12	5	300	6	0.020	0.466	0.000	1.016
CD	0	LC	0.2	12	15	100	3	0.339	1.469	0.000	1.192
CD	0	LC	0.2	12	15	100	6	0.083	0.893	0.000	1.182
CD	0	LC	0.2	12	15	300	3	0.294	1.173	0.000	1.094
CD	0	LC	0.2	12	15	300	6	0.094	0.753	0.000	1.097
CD	0	LC	0.7	6	5	100	3	0.024	0.648	0.000	0.973
CD	0	LC	0.7	6	5	100	6	0.007	0.909	0.000	0.968
CD	0	LC	0.7	6	5	300	3	0.024	0.444	0.000	1.010
CD	0	LC	0.7	6	5	300	6	0.083	0.569	0.000	0.990

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	0	LC	0.7	6	15	100	3	0.182	1.173	0.000	1.123
CD	0	LC	0.7	6	15	100	6	0.055	1.377	0.000	1.032
CD	0	LC	0.7	6	15	300	3	0.139	0.803	0.000	1.087
CD	0	LC	0.7	6	15	300	6	0.086	0.912	0.000	1.032
CD	0	LC	0.7	12	5	100	3	0.029	0.845	0.000	1.007
CD	0	LC	0.7	12	5	100	6	0.028	0.918	0.000	0.953
CD	0	LC	0.7	12	5	300	3	0.036	0.731	0.000	1.006
CD	0	LC	0.7	12	5	300	6	0.048	0.674	0.000	0.986
CD	0	LC	0.7	12	15	100	3	0.366	1.777	0.000	1.166
CD	0	LC	0.7	12	15	100	6	0.290	1.544	0.000	1.047
CD	0	LC	0.7	12	15	300	3	0.251	1.288	0.000	1.085
CD	0	LC	0.7	12	15	300	6	0.102	1.085	0.000	1.035
CD	0	RC	0.2	6	5	100	3	0.023	0.435	0.000	1.017
CD	0	RC	0.2	6	5	100	6	0.009	0.468	0.000	0.984
CD	0	RC	0.2	6	5	300	3	0.028	0.358	0.000	0.995
CD	0	RC	0.2	6	5	300	6	0.019	0.321	0.000	1.034
CD	0	RC	0.2	6	15	100	3	0.204	0.818	0.000	1.200
CD	0	RC	0.2	6	15	100	6	0.039	0.711	0.000	1.179
CD	0	RC	0.2	6	15	300	3	0.134	0.618	0.000	1.066
CD	0	RC	0.2	6	15	300	6	0.067	0.519	0.000	1.118

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	0	RC	0.2	12	5	100	3	0.034	0.627	0.000	1.021
CD	0	RC	0.2	12	5	100	6	0.027	0.586	0.000	0.992
CD	0	RC	0.2	12	5	300	3	0.041	0.640	0.000	1.015
CD	0	RC	0.2	12	5	300	6	0.036	0.478	0.000	1.016
CD	0	RC	0.2	12	15	100	3	0.412	1.308	0.000	1.198
CD	0	RC	0.2	12	15	100	6	0.141	0.937	0.000	1.195
CD	0	RC	0.2	12	15	300	3	0.236	1.116	0.000	1.086
CD	0	RC	0.2	12	15	300	6	0.161	0.803	0.000	1.121
CD	0	RC	0.7	6	5	100	3	0.008	0.548	0.000	0.964
CD	0	RC	0.7	6	5	100	6	0.004	0.859	0.000	0.961
CD	0	RC	0.7	6	5	300	3	0.023	0.469	0.000	1.014
CD	0	RC	0.7	6	5	300	6	0.188	0.566	0.000	0.990
CD	0	RC	0.7	6	15	100	3	0.136	1.085	0.000	1.085
CD	0	RC	0.7	6	15	100	6	0.017	1.331	0.000	1.031
CD	0	RC	0.7	6	15	300	3	0.132	0.819	0.000	1.093
CD	0	RC	0.7	6	15	300	6	0.266	0.884	0.000	1.025
CD	0	RC	0.7	12	5	100	3	0.041	0.886	0.000	1.008
CD	0	RC	0.7	12	5	100	6	0.019	0.947	0.000	0.956
CD	0	RC	0.7	12	5	300	3	0.076	0.746	0.000	1.006
CD	0	RC	0.7	12	5	300	6	0.085	0.703	0.000	0.986

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	0	RC	0.7	12	15	100	3	0.336	1.736	0.000	1.162
CD	0	RC	0.7	12	15	100	6	0.095	1.579	0.000	1.046
CD	0	RC	0.7	12	15	300	3	0.325	1.318	0.000	1.079
CD	0	RC	0.7	12	15	300	6	0.219	1.097	0.000	1.038
CD	0	RM	0.2	6	5	100	3	0.001	0.156	0.000	0.996
CD	0	RM	0.2	6	5	100	6	0.001	0.160	0.000	0.965
CD	0	RM	0.2	6	5	300	3	0.002	0.060	0.000	0.979
CD	0	RM	0.2	6	5	300	6	0.002	0.069	0.000	1.019
CD	0	RM	0.2	6	15	100	3	0.005	0.393	0.000	1.054
CD	0	RM	0.2	6	15	100	6	0.005	0.374	0.000	1.140
CD	0	RM	0.2	6	15	300	3	0.003	0.113	0.000	0.989
CD	0	RM	0.2	6	15	300	6	0.009	0.171	0.000	1.067
CD	0	RM	0.2	12	5	100	3	0.000	0.207	0.000	1.002
CD	0	RM	0.2	12	5	100	6	0.002	0.243	0.000	0.974
CD	0	RM	0.2	12	5	300	3	0.001	0.119	0.000	0.999
CD	0	RM	0.2	12	5	300	6	0.006	0.119	0.000	0.999
CD	0	RM	0.2	12	15	100	3	0.010	0.671	0.000	1.079
CD	0	RM	0.2	12	15	100	6	0.007	0.608	0.000	1.138
CD	0	RM	0.2	12	15	300	3	0.001	0.262	0.000	1.008
CD	0	RM	0.2	12	15	300	6	0.007	0.251	0.000	1.046

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	0	RM	0.7	6	5	100	3	0.002	0.155	0.000	0.954
CD	0	RM	0.7	6	5	100	6	0.012	0.212	0.000	0.965
CD	0	RM	0.7	6	5	300	3	0.002	0.081	0.000	0.999
CD	0	RM	0.7	6	5	300	6	0.119	0.092	0.000	0.988
CD	0	RM	0.7	6	15	100	3	0.020	0.376	0.000	1.006
CD	0	RM	0.7	6	15	100	6	0.584	0.546	0.000	1.136
CD	0	RM	0.7	6	15	300	3	0.000	0.144	0.000	1.010
CD	0	RM	0.7	6	15	300	6	0.057	0.232	0.000	1.037
CD	0	RM	0.7	12	5	100	3	0.001	0.204	0.000	0.992
CD	0	RM	0.7	12	5	100	6	0.005	0.264	0.000	0.960
CD	0	RM	0.7	12	5	300	3	0.028	0.121	0.000	0.990
CD	0	RM	0.7	12	5	300	6	0.188	0.142	0.000	0.982
CD	0	RM	0.7	12	15	100	3	0.009	0.642	0.000	1.056
CD	0	RM	0.7	12	15	100	6	0.046	0.687	0.000	1.145
CD	0	RM	0.7	12	15	300	3	0.014	0.253	0.000	1.001
CD	0	RM	0.7	12	15	300	6	0.308	0.320	0.000	1.027
CD	2	LC	0.2	6	5	100	3	0.028	0.328	0.000	0.982
CD	2	LC	0.2	6	5	100	6	0.008	0.206	0.000	0.953
CD	2	LC	0.2	6	5	300	3	0.027	0.320	0.000	0.995
CD	2	LC	0.2	6	5	300	6	0.017	0.200	0.000	0.994

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	2	LC	0.2	6	15	100	3	0.144	0.571	0.000	1.067
CD	2	LC	0.2	6	15	100	6	0.034	0.344	0.000	1.052
CD	2	LC	0.2	6	15	300	3	0.135	0.548	0.000	1.064
CD	2	LC	0.2	6	15	300	6	0.070	0.305	0.000	1.042
CD	2	LC	0.2	12	5	100	3	0.052	0.618	0.000	0.962
CD	2	LC	0.2	12	5	100	6	0.046	0.369	0.000	0.971
CD	2	LC	0.2	12	5	300	3	0.051	0.584	0.000	1.025
CD	2	LC	0.2	12	5	300	6	0.021	0.368	0.000	1.027
CD	2	LC	0.2	12	15	100	3	0.267	1.096	0.000	1.045
CD	2	LC	0.2	12	15	100	6	0.171	0.600	0.000	1.095
CD	2	LC	0.2	12	15	300	3	0.275	1.090	0.000	1.099
CD	2	LC	0.2	12	15	300	6	0.126	0.606	0.000	1.103
CD	2	LC	0.7	6	5	100	3	0.020	0.301	0.000	0.973
CD	2	LC	0.7	6	5	100	6	0.026	0.289	0.000	0.977
CD	2	LC	0.7	6	5	300	3	0.034	0.388	0.000	1.025
CD	2	LC	0.7	6	5	300	6	0.019	0.248	0.000	0.976
CD	2	LC	0.7	6	15	100	3	0.093	0.553	0.000	1.036
CD	2	LC	0.7	6	15	100	6	0.041	0.453	0.000	1.036
CD	2	LC	0.7	6	15	300	3	0.129	0.615	0.000	1.092
CD	2	LC	0.7	6	15	300	6	0.044	0.379	0.000	1.004

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	2	LC	0.7	12	5	100	3	0.014	0.562	0.000	0.932
CD	2	LC	0.7	12	5	100	6	0.025	0.427	0.000	0.972
CD	2	LC	0.7	12	5	300	3	0.047	0.678	0.000	1.017
CD	2	LC	0.7	12	5	300	6	0.028	0.435	0.000	0.977
CD	2	LC	0.7	12	15	100	3	0.229	1.090	0.000	1.009
CD	2	LC	0.7	12	15	100	6	0.280	0.692	0.000	1.052
CD	2	LC	0.7	12	15	300	3	0.247	1.175	0.000	1.085
CD	2	LC	0.7	12	15	300	6	0.141	0.682	0.000	1.019
CD	2	RC	0.2	6	5	100	3	0.041	0.464	0.000	1.025
CD	2	RC	0.2	6	5	100	6	0.018	0.440	0.000	1.034
CD	2	RC	0.2	6	5	300	3	0.029	0.453	0.000	1.007
CD	2	RC	0.2	6	5	300	6	0.008	0.467	0.000	1.020
CD	2	RC	0.2	6	15	100	3	0.103	0.706	0.000	1.195
CD	2	RC	0.2	6	15	100	6	0.095	0.620	0.000	1.455
CD	2	RC	0.2	6	15	300	3	0.149	0.714	0.000	1.092
CD	2	RC	0.2	6	15	300	6	0.055	0.667	0.000	1.156
CD	2	RC	0.2	12	5	100	3	0.060	0.653	0.000	0.990
CD	2	RC	0.2	12	5	100	6	0.019	0.578	0.000	1.042
CD	2	RC	0.2	12	5	300	3	0.035	0.701	0.000	1.033
CD	2	RC	0.2	12	5	300	6	0.024	0.579	0.000	1.048

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	2	RC	0.2	12	15	100	3	0.373	1.136	0.000	1.157
CD	2	RC	0.2	12	15	100	6	0.154	0.857	0.000	1.423
CD	2	RC	0.2	12	15	300	3	0.234	1.206	0.000	1.131
CD	2	RC	0.2	12	15	300	6	0.133	0.898	0.000	1.191
CD	2	RC	0.7	6	5	100	3	0.025	0.637	0.000	1.006
CD	2	RC	0.7	6	5	100	6	0.153	0.917	0.000	1.175
CD	2	RC	0.7	6	5	300	3	0.028	0.635	0.000	1.011
CD	2	RC	0.7	6	5	300	6	0.004	0.921	0.000	0.996
CD	2	RC	0.7	6	15	100	3	0.128	0.992	0.000	1.160
CD	2	RC	0.7	6	15	100	6	0.060	1.244	0.000	1.404
CD	2	RC	0.7	6	15	300	3	0.137	0.966	0.000	1.100
CD	2	RC	0.7	6	15	300	6	0.150	1.243	0.000	1.077
CD	2	RC	0.7	12	5	100	3	0.049	0.820	0.000	0.964
CD	2	RC	0.7	12	5	100	6	1.535	1.120	0.000	1.061
CD	2	RC	0.7	12	5	300	3	0.056	0.895	0.000	1.029
CD	2	RC	0.7	12	5	300	6	0.040	1.067	0.000	0.998
CD	2	RC	0.7	12	15	100	3	0.272	1.401	0.000	1.124
CD	2	RC	0.7	12	15	100	6	1.012	1.544	0.000	1.298
CD	2	RC	0.7	12	15	300	3	0.291	1.442	0.000	1.125
CD	2	RC	0.7	12	15	300	6	0.116	1.478	0.000	1.096

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	2	RM	0.2	6	5	100	3	0.001	0.127	0.000	0.971
CD	2	RM	0.2	6	5	100	6	0.007	0.143	0.000	0.950
CD	2	RM	0.2	6	5	300	3	0.001	0.074	0.000	0.981
CD	2	RM	0.2	6	5	300	6	0.011	0.098	0.000	0.989
CD	2	RM	0.2	6	15	100	3	0.004	0.245	0.000	1.004
CD	2	RM	0.2	6	15	100	6	0.005	0.307	0.000	1.154
CD	2	RM	0.2	6	15	300	3	0.001	0.148	0.000	0.991
CD	2	RM	0.2	6	15	300	6	0.018	0.204	0.000	1.036
CD	2	RM	0.2	12	5	100	3	0.002	0.212	0.000	0.947
CD	2	RM	0.2	12	5	100	6	0.005	0.218	0.000	0.970
CD	2	RM	0.2	12	5	300	3	0.001	0.121	0.000	1.012
CD	2	RM	0.2	12	5	300	6	0.001	0.117	0.000	1.016
CD	2	RM	0.2	12	15	100	3	0.018	0.418	0.000	0.993
CD	2	RM	0.2	12	15	100	6	0.020	0.479	0.000	1.179
CD	2	RM	0.2	12	15	300	3	0.005	0.227	0.000	1.021
CD	2	RM	0.2	12	15	300	6	0.007	0.308	0.000	1.071
CD	2	RM	0.7	6	5	100	3	0.001	0.135	0.000	0.966
CD	2	RM	0.7	6	5	100	6	0.255	0.258	0.000	1.003
CD	2	RM	0.7	6	5	300	3	0.015	0.084	0.000	1.013
CD	2	RM	0.7	6	5	300	6	0.005	0.124	0.000	0.975

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
CD	2	RM	0.7	6	15	100	3	0.004	0.268	0.000	1.001
CD	2	RM	0.7	6	15	100	6	0.363	0.454	0.000	1.298
CD	2	RM	0.7	6	15	300	3	0.009	0.187	0.000	1.021
CD	2	RM	0.7	6	15	300	6	0.095	0.297	0.000	1.027
CD	2	RM	0.7	12	5	100	3	0.005	0.176	0.000	0.926
CD	2	RM	0.7	12	5	100	6	0.012	0.275	0.000	0.991
CD	2	RM	0.7	12	5	300	3	0.002	0.139	0.000	1.003
CD	2	RM	0.7	12	5	300	6	0.003	0.180	0.000	0.974
CD	2	RM	0.7	12	15	100	3	0.077	0.456	0.000	0.965
CD	2	RM	0.7	12	15	100	6	0.229	0.745	0.000	1.219
CD	2	RM	0.7	12	15	300	3	0.005	0.252	0.000	1.012
CD	2	RM	0.7	12	15	300	6	0.099	0.408	0.000	1.032
EM	0	LC	0.2	6	5	100	3	0.009	0.452	2.323	1.009
EM	0	LC	0.2	6	5	100	6	0.004	0.331	0.699	0.967
EM	0	LC	0.2	6	5	300	3	0.021	0.339	2.199	0.985
EM	0	LC	0.2	6	5	300	6	0.008	0.261	0.741	1.018
EM	0	LC	0.2	6	15	100	3	0.032	0.807	2.486	1.073
EM	0	LC	0.2	6	15	100	6	0.019	0.646	0.827	1.030
EM	0	LC	0.2	6	15	300	3	0.068	0.611	1.966	1.015
EM	0	LC	0.2	6	15	300	6	0.017	0.512	0.850	1.044

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	0	LC	0.2	12	5	100	3	0.017	0.666	2.349	1.017
EM	0	LC	0.2	12	5	100	6	0.004	0.502	0.756	0.970
EM	0	LC	0.2	12	5	300	3	0.034	0.643	2.330	1.004
EM	0	LC	0.2	12	5	300	6	0.002	0.441	0.808	0.999
EM	0	LC	0.2	12	15	100	3	0.116	1.378	2.767	1.084
EM	0	LC	0.2	12	15	100	6	0.009	0.932	0.878	1.028
EM	0	LC	0.2	12	15	300	3	0.137	1.142	1.956	1.039
EM	0	LC	0.2	12	15	300	6	0.021	0.828	0.874	1.023
EM	0	LC	0.7	6	5	100	3	0.008	0.469	1.744	0.965
EM	0	LC	0.7	6	5	100	6	0.010	0.434	0.218	0.954
EM	0	LC	0.7	6	5	300	3	0.012	0.355	1.820	1.003
EM	0	LC	0.7	6	5	300	6	0.032	0.341	0.233	0.986
EM	0	LC	0.7	6	15	100	3	0.007	0.650	1.497	0.995
EM	0	LC	0.7	6	15	100	6	0.042	0.767	0.292	0.984
EM	0	LC	0.7	6	15	300	3	0.031	0.573	1.256	1.018
EM	0	LC	0.7	6	15	300	6	0.072	0.704	0.317	1.017
EM	0	LC	0.7	12	5	100	3	0.013	0.727	1.780	1.003
EM	0	LC	0.7	12	5	100	6	0.028	0.538	0.244	0.944
EM	0	LC	0.7	12	5	300	3	0.016	0.664	2.016	1.003
EM	0	LC	0.7	12	5	300	6	0.019	0.501	0.261	0.983

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	0	LC	0.7	12	15	100	3	0.065	1.241	1.848	1.055
EM	0	LC	0.7	12	15	100	6	0.169	1.052	0.356	0.975
EM	0	LC	0.7	12	15	300	3	0.070	1.035	1.479	1.027
EM	0	LC	0.7	12	15	300	6	0.082	1.014	0.334	1.027
EM	0	RC	0.2	6	5	100	3	0.009	0.421	2.349	1.010
EM	0	RC	0.2	6	5	100	6	0.003	0.352	0.716	0.965
EM	0	RC	0.2	6	5	300	3	0.015	0.342	2.313	0.985
EM	0	RC	0.2	6	5	300	6	0.007	0.272	0.768	1.021
EM	0	RC	0.2	6	15	100	3	0.059	0.826	2.585	1.081
EM	0	RC	0.2	6	15	100	6	0.015	0.694	0.828	1.041
EM	0	RC	0.2	6	15	300	3	0.058	0.613	1.942	1.011
EM	0	RC	0.2	6	15	300	6	0.018	0.535	0.910	1.045
EM	0	RC	0.2	12	5	100	3	0.011	0.630	2.402	1.015
EM	0	RC	0.2	12	5	100	6	0.006	0.490	0.760	0.968
EM	0	RC	0.2	12	5	300	3	0.019	0.626	2.253	1.004
EM	0	RC	0.2	12	5	300	6	0.011	0.457	0.787	1.000
EM	0	RC	0.2	12	15	100	3	0.133	1.353	2.723	1.088
EM	0	RC	0.2	12	15	100	6	0.024	0.943	0.887	1.017
EM	0	RC	0.2	12	15	300	3	0.112	1.143	1.923	1.039
EM	0	RC	0.2	12	15	300	6	0.050	0.859	0.894	1.032

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	0	RC	0.7	6	5	100	3	0.004	0.409	1.608	0.963
EM	0	RC	0.7	6	5	100	6	0.009	0.381	0.204	0.951
EM	0	RC	0.7	6	5	300	3	0.009	0.362	1.506	1.006
EM	0	RC	0.7	6	5	300	6	0.169	0.329	0.207	0.986
EM	0	RC	0.7	6	15	100	3	0.008	0.682	1.652	0.989
EM	0	RC	0.7	6	15	100	6	0.013	0.750	0.262	0.983
EM	0	RC	0.7	6	15	300	3	0.021	0.559	1.238	1.019
EM	0	RC	0.7	6	15	300	6	0.383	0.696	0.314	1.018
EM	0	RC	0.7	12	5	100	3	0.017	0.727	1.602	1.002
EM	0	RC	0.7	12	5	100	6	0.093	0.520	0.230	0.943
EM	0	RC	0.7	12	5	300	3	0.048	0.681	2.018	1.003
EM	0	RC	0.7	12	5	300	6	0.046	0.507	0.261	0.983
EM	0	RC	0.7	12	15	100	3	0.104	1.157	1.717	1.048
EM	0	RC	0.7	12	15	100	6	0.009	0.974	0.320	0.978
EM	0	RC	0.7	12	15	300	3	0.137	1.077	1.436	1.026
EM	0	RC	0.7	12	15	300	6	0.087	1.006	0.372	1.024
EM	0	RM	0.2	6	5	100	3	0.000	0.087	0.874	0.995
EM	0	RM	0.2	6	5	100	6	0.000	0.056	0.129	0.942
EM	0	RM	0.2	6	5	300	3	0.003	0.040	0.926	0.977
EM	0	RM	0.2	6	5	300	6	0.003	0.028	0.130	1.012

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	0	RM	0.2	6	15	100	3	0.000	0.149	0.614	1.012
EM	0	RM	0.2	6	15	100	6	0.001	0.103	0.149	0.967
EM	0	RM	0.2	6	15	300	3	0.005	0.069	0.450	0.980
EM	0	RM	0.2	6	15	300	6	0.005	0.048	0.144	1.019
EM	0	RM	0.2	12	5	100	3	0.000	0.104	0.830	1.003
EM	0	RM	0.2	12	5	100	6	0.001	0.079	0.144	0.947
EM	0	RM	0.2	12	5	300	3	0.001	0.073	0.797	0.994
EM	0	RM	0.2	12	5	300	6	0.003	0.045	0.136	0.991
EM	0	RM	0.2	12	15	100	3	0.000	0.276	0.592	1.022
EM	0	RM	0.2	12	15	100	6	0.002	0.154	0.161	0.966
EM	0	RM	0.2	12	15	300	3	0.000	0.118	0.478	0.997
EM	0	RM	0.2	12	15	300	6	0.003	0.080	0.153	0.997
EM	0	RM	0.7	6	5	100	3	0.004	0.079	0.716	0.955
EM	0	RM	0.7	6	5	100	6	0.004	0.059	0.022	0.944
EM	0	RM	0.7	6	5	300	3	0.002	0.049	0.698	0.999
EM	0	RM	0.7	6	5	300	6	0.032	0.024	0.023	0.981
EM	0	RM	0.7	6	15	100	3	0.031	0.140	0.413	0.976
EM	0	RM	0.7	6	15	100	6	0.388	0.126	0.035	0.989
EM	0	RM	0.7	6	15	300	3	0.002	0.082	0.350	1.005
EM	0	RM	0.7	6	15	300	6	0.280	0.057	0.034	0.994

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	0	RM	0.7	12	5	100	3	0.000	0.143	0.717	0.989
EM	0	RM	0.7	12	5	100	6	0.031	0.081	0.025	0.935
EM	0	RM	0.7	12	5	300	3	0.019	0.072	0.703	0.993
EM	0	RM	0.7	12	5	300	6	0.104	0.040	0.026	0.974
EM	0	RM	0.7	12	15	100	3	0.004	0.298	0.431	1.012
EM	0	RM	0.7	12	15	100	6	0.063	0.148	0.039	0.965
EM	0	RM	0.7	12	15	300	3	0.016	0.112	0.355	0.997
EM	0	RM	0.7	12	15	300	6	0.313	0.081	0.038	0.984
EM	2	LC	0.2	6	5	100	3	0.016	0.356	1.287	0.983
EM	2	LC	0.2	6	5	100	6	0.004	0.214	0.249	0.933
EM	2	LC	0.2	6	5	300	3	0.016	0.331	1.288	0.985
EM	2	LC	0.2	6	5	300	6	0.011	0.225	0.229	0.985
EM	2	LC	0.2	6	15	100	3	0.089	0.632	1.012	1.031
EM	2	LC	0.2	6	15	100	6	0.016	0.443	0.300	0.965
EM	2	LC	0.2	6	15	300	3	0.075	0.598	0.942	1.019
EM	2	LC	0.2	6	15	300	6	0.037	0.454	0.282	1.005
EM	2	LC	0.2	12	5	100	3	0.034	0.607	1.373	0.959
EM	2	LC	0.2	12	5	100	6	0.028	0.387	0.259	0.950
EM	2	LC	0.2	12	5	300	3	0.033	0.630	1.300	1.024
EM	2	LC	0.2	12	5	300	6	0.008	0.414	0.277	1.015

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	2	LC	0.2	12	15	100	3	0.184	1.150	1.058	1.012
EM	2	LC	0.2	12	15	100	6	0.095	0.771	0.340	0.997
EM	2	LC	0.2	12	15	300	3	0.172	1.182	0.971	1.066
EM	2	LC	0.2	12	15	300	6	0.053	0.777	0.336	1.048
EM	2	LC	0.7	6	5	100	3	0.013	0.308	1.112	0.976
EM	2	LC	0.7	6	5	100	6	0.033	0.282	0.096	0.975
EM	2	LC	0.7	6	5	300	3	0.027	0.385	17.12	1.024
EM	2	LC	0.7	6	5	300	6	0.026	0.257	0.097	0.976
EM	2	LC	0.7	6	15	100	3	0.051	0.544	0.724	1.012
EM	2	LC	0.7	6	15	100	6	0.068	0.576	0.124	1.033
EM	2	LC	0.7	6	15	300	3	0.068	0.592	5.256	1.054
EM	2	LC	0.7	6	15	300	6	0.051	0.529	0.141	1.014
EM	2	LC	0.7	12	5	100	3	0.031	0.579	1.135	0.935
EM	2	LC	0.7	12	5	100	6	0.027	0.470	0.133	0.966
EM	2	LC	0.7	12	5	300	3	0.034	0.677	1.156	1.012
EM	2	LC	0.7	12	5	300	6	0.023	0.472	0.125	0.977
EM	2	LC	0.7	12	15	100	3	0.138	0.966	0.840	0.980
EM	2	LC	0.7	12	15	100	6	0.238	0.911	0.160	1.031
EM	2	LC	0.7	12	15	300	3	0.141	1.102	0.799	1.049
EM	2	LC	0.7	12	15	300	6	0.130	0.903	0.166	1.029

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	2	RC	0.2	6	5	100	3	0.020	0.414	5.337	1.019
EM	2	RC	0.2	6	5	100	6	0.010	0.321	2.404	1.005
EM	2	RC	0.2	6	5	300	3	0.008	0.398	5.856	0.992
EM	2	RC	0.2	6	5	300	6	0.010	0.356	2.607	1.015
EM	2	RC	0.2	6	15	100	3	0.024	0.675	5.592	1.111
EM	2	RC	0.2	6	15	100	6	0.019	0.596	2.826	1.136
EM	2	RC	0.2	6	15	300	3	0.032	0.653	5.639	1.022
EM	2	RC	0.2	6	15	300	6	0.016	0.639	2.839	1.053
EM	2	RC	0.2	12	5	100	3	0.016	0.608	5.558	0.981
EM	2	RC	0.2	12	5	100	6	0.002	0.463	2.550	1.008
EM	2	RC	0.2	12	5	300	3	0.007	0.674	6.448	1.028
EM	2	RC	0.2	12	5	300	6	0.007	0.505	2.632	1.028
EM	2	RC	0.2	12	15	100	3	0.163	1.079	5.170	1.071
EM	2	RC	0.2	12	15	100	6	0.028	0.874	2.804	1.125
EM	2	RC	0.2	12	15	300	3	0.092	1.199	5.900	1.076
EM	2	RC	0.2	12	15	300	6	0.008	0.911	2.982	1.065
EM	2	RC	0.7	6	5	100	3	0.002	0.408	3.555	0.995
EM	2	RC	0.7	6	5	100	6	0.079	0.550	10.06	1.147
EM	2	RC	0.7	6	5	300	3	0.001	0.407	3.976	1.003
EM	2	RC	0.7	6	5	300	6	0.005	0.496	0.922	0.986

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	2	RC	0.7	6	15	100	3	0.024	0.611	3.874	1.110
EM	2	RC	0.7	6	15	100	6	0.030	0.834	4.457	1.227
EM	2	RC	0.7	6	15	300	3	0.015	0.566	4.307	1.036
EM	2	RC	0.7	6	15	300	6	0.127	0.874	0.913	1.030
EM	2	RC	0.7	12	5	100	3	0.024	0.619	1.488	0.954
EM	2	RC	0.7	12	5	100	6	1.198	0.683	0.983	1.028
EM	2	RC	0.7	12	5	300	3	0.018	0.699	4.515	1.015
EM	2	RC	0.7	12	5	300	6	0.032	0.705	0.972	0.986
EM	2	RC	0.7	12	15	100	3	0.040	0.914	1.578	1.061
EM	2	RC	0.7	12	15	100	6	0.118	1.280	0.997	1.113
EM	2	RC	0.7	12	15	300	3	0.048	0.978	4.582	1.044
EM	2	RC	0.7	12	15	300	6	0.070	1.202	0.965	1.034
EM	2	RM	0.2	6	5	100	3	0.000	0.070	0.844	0.974
EM	2	RM	0.2	6	5	100	6	0.005	0.048	0.155	0.925
EM	2	RM	0.2	6	5	300	3	0.001	0.042	0.801	0.976
EM	2	RM	0.2	6	5	300	6	0.004	0.028	0.143	0.981
EM	2	RM	0.2	6	15	100	3	0.002	0.127	0.492	0.985
EM	2	RM	0.2	6	15	100	6	0.005	0.097	0.188	0.959
EM	2	RM	0.2	6	15	300	3	0.001	0.067	0.465	0.980
EM	2	RM	0.2	6	15	300	6	0.005	0.050	0.151	0.987

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	2	RM	0.2	12	5	100	3	0.001	0.112	0.796	0.946
EM	2	RM	0.2	12	5	100	6	0.003	0.069	0.140	0.942
EM	2	RM	0.2	12	5	300	3	0.000	0.071	0.976	1.015
EM	2	RM	0.2	12	5	300	6	0.000	0.034	0.146	1.007
EM	2	RM	0.2	12	15	100	3	0.008	0.211	0.516	0.961
EM	2	RM	0.2	12	15	100	6	0.003	0.139	0.164	0.968
EM	2	RM	0.2	12	15	300	3	0.001	0.109	0.521	1.019
EM	2	RM	0.2	12	15	300	6	0.002	0.077	0.154	1.015
EM	2	RM	0.7	6	5	100	3	0.001	0.054	0.644	0.968
EM	2	RM	0.7	6	5	100	6	0.200	0.061	0.031	0.970
EM	2	RM	0.7	6	5	300	3	0.013	0.040	0.668	1.011
EM	2	RM	0.7	6	5	300	6	0.006	0.026	0.028	0.968
EM	2	RM	0.7	6	15	100	3	0.009	0.113	0.331	0.980
EM	2	RM	0.7	6	15	100	6	0.579	0.099	0.039	1.020
EM	2	RM	0.7	6	15	300	3	0.013	0.060	0.332	1.014
EM	2	RM	0.7	6	15	300	6	0.090	0.046	0.037	0.979
EM	2	RM	0.7	12	5	100	3	0.001	0.102	0.734	0.925
EM	2	RM	0.7	12	5	100	6	0.001	0.077	0.034	0.956
EM	2	RM	0.7	12	5	300	3	0.002	0.061	0.689	1.000
EM	2	RM	0.7	12	5	300	6	0.003	0.040	0.032	0.965

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
EM	2	RM	0.7	12	15	100	3	0.031	0.174	0.359	0.935
EM	2	RM	0.7	12	15	100	6	0.020	0.134	0.050	1.004
EM	2	RM	0.7	12	15	300	3	0.001	0.098	0.341	1.003
EM	2	RM	0.7	12	15	300	6	0.057	0.071	0.045	0.977
MR	0	LC	0.2	6	5	100	3	0.023	0.559	4.761	1.015
MR	0	LC	0.2	6	5	100	6	0.007	0.454	4.599	0.983
MR	0	LC	0.2	6	5	300	3	0.030	0.421	4.757	0.990
MR	0	LC	0.2	6	5	300	6	0.013	0.336	4.772	1.024
MR	0	LC	0.2	6	15	100	3	0.127	1.077	4.208	1.098
MR	0	LC	0.2	6	15	100	6	0.023	0.855	3.468	1.045
MR	0	LC	0.2	6	15	300	3	0.108	0.785	3.469	1.029
MR	0	LC	0.2	6	15	300	6	0.036	0.658	3.496	1.057
MR	0	LC	0.2	12	5	100	3	0.030	0.799	4.584	1.019
MR	0	LC	0.2	12	5	100	6	0.009	0.630	4.566	0.976
MR	0	LC	0.2	12	5	300	3	0.045	0.798	4.823	1.011
MR	0	LC	0.2	12	5	300	6	0.008	0.550	4.736	1.003
MR	0	LC	0.2	12	15	100	3	0.210	1.802	4.211	1.095
MR	0	LC	0.2	12	15	100	6	0.029	1.189	3.459	1.032
MR	0	LC	0.2	12	15	300	3	0.194	1.484	3.509	1.053
MR	0	LC	0.2	12	15	300	6	0.044	1.055	3.481	1.036

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	0	LC	0.7	6	5	100	3	0.021	0.619	4.542	0.974
MR	0	LC	0.7	6	5	100	6	0.179	0.719	4.593	0.988
MR	0	LC	0.7	6	5	300	3	0.013	0.454	4.637	1.006
MR	0	LC	0.7	6	5	300	6	0.028	0.529	4.662	0.993
MR	0	LC	0.7	6	15	100	3	0.062	1.010	3.893	1.034
MR	0	LC	0.7	6	15	100	6	0.228	1.223	3.358	1.032
MR	0	LC	0.7	6	15	300	3	0.066	0.802	3.323	1.038
MR	0	LC	0.7	6	15	300	6	0.056	1.040	3.330	1.027
MR	0	LC	0.7	12	5	100	3	0.019	0.882	4.567	1.010
MR	0	LC	0.7	12	5	100	6	0.025	0.807	4.473	0.969
MR	0	LC	0.7	12	5	300	3	0.027	0.803	4.678	1.000
MR	0	LC	0.7	12	5	300	6	0.090	0.679	4.618	0.987
MR	0	LC	0.7	12	15	100	3	0.146	1.745	4.053	1.075
MR	0	LC	0.7	12	15	100	6	0.041	1.514	3.319	1.003
MR	0	LC	0.7	12	15	300	3	0.122	1.376	3.326	1.037
MR	0	LC	0.7	12	15	300	6	0.088	1.367	3.298	1.020
MR	0	RC	0.2	6	5	100	3	0.021	0.520	4.579	1.013
MR	0	RC	0.2	6	5	100	6	0.009	0.474	4.640	0.986
MR	0	RC	0.2	6	5	300	3	0.026	0.428	4.756	0.990
MR	0	RC	0.2	6	5	300	6	0.014	0.347	4.821	1.027

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	0	RC	0.2	6	15	100	3	0.137	1.091	4.185	1.108
MR	0	RC	0.2	6	15	100	6	0.029	0.887	3.466	1.063
MR	0	RC	0.2	6	15	300	3	0.103	0.791	3.456	1.028
MR	0	RC	0.2	6	15	300	6	0.042	0.677	3.507	1.059
MR	0	RC	0.2	12	5	100	3	0.019	0.763	4.714	1.013
MR	0	RC	0.2	12	5	100	6	0.012	0.627	4.610	0.977
MR	0	RC	0.2	12	5	300	3	0.025	0.776	4.726	1.008
MR	0	RC	0.2	12	5	300	6	0.019	0.564	4.778	1.005
MR	0	RC	0.2	12	15	100	3	0.210	1.774	4.254	1.094
MR	0	RC	0.2	12	15	100	6	0.056	1.201	3.460	1.029
MR	0	RC	0.2	12	15	300	3	0.155	1.478	3.478	1.050
MR	0	RC	0.2	12	15	300	6	0.084	1.082	3.511	1.046
MR	0	RC	0.7	6	5	100	3	0.008	0.564	4.510	0.930
MR	0	RC	0.7	6	5	100	6	0.196	0.680	4.518	0.989
MR	0	RC	0.7	6	5	300	3	0.015	0.463	4.638	1.009
MR	0	RC	0.7	6	5	300	6	0.085	0.518	4.599	0.994
MR	0	RC	0.7	6	15	100	3	0.031	1.039	3.978	1.010
MR	0	RC	0.7	6	15	100	6	0.106	1.208	3.335	1.038
MR	0	RC	0.7	6	15	300	3	0.062	0.787	3.275	1.040
MR	0	RC	0.7	6	15	300	6	0.086	1.028	3.312	1.026

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	0	RC	0.7	12	5	100	3	0.019	0.881	4.553	1.004
MR	0	RC	0.7	12	5	100	6	0.017	0.813	4.688	0.969
MR	0	RC	0.7	12	5	300	3	0.056	0.827	4.751	1.003
MR	0	RC	0.7	12	5	300	6	0.045	0.693	4.730	0.988
MR	0	RC	0.7	12	15	100	3	0.154	1.683	3.956	1.066
MR	0	RC	0.7	12	15	100	6	0.037	1.464	3.428	1.010
MR	0	RC	0.7	12	15	300	3	0.180	1.425	3.382	1.037
MR	0	RC	0.7	12	15	300	6	0.081	1.369	3.354	1.021
MR	0	RM	0.2	6	5	100	3	0.002	0.142	0.996	0.992
MR	0	RM	0.2	6	5	100	6	0.002	0.120	0.975	0.949
MR	0	RM	0.2	6	5	300	3	0.004	0.096	1.004	0.978
MR	0	RM	0.2	6	5	300	6	0.004	0.082	0.991	1.013
MR	0	RM	0.2	6	15	100	3	0.016	0.472	1.336	1.013
MR	0	RM	0.2	6	15	100	6	0.005	0.309	1.011	0.978
MR	0	RM	0.2	6	15	300	3	0.015	0.275	0.989	0.984
MR	0	RM	0.2	6	15	300	6	0.013	0.236	0.994	1.025
MR	0	RM	0.2	12	5	100	3	0.001	0.160	0.981	0.999
MR	0	RM	0.2	12	5	100	6	0.001	0.149	1.002	0.949
MR	0	RM	0.2	12	5	300	3	0.003	0.182	1.004	0.998
MR	0	RM	0.2	12	5	300	6	0.003	0.127	0.988	0.991

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	0	RM	0.2	12	15	100	3	0.019	0.773	1.337	1.016
MR	0	RM	0.2	12	15	100	6	0.008	0.418	1.004	0.972
MR	0	RM	0.2	12	15	300	3	0.011	0.520	1.006	1.001
MR	0	RM	0.2	12	15	300	6	0.008	0.356	0.992	1.000
MR	0	RM	0.7	6	5	100	3	0.001	0.154	0.991	0.952
MR	0	RM	0.7	6	5	100	6	0.079	0.199	1.017	0.965
MR	0	RM	0.7	6	5	300	3	0.002	0.115	1.017	0.999
MR	0	RM	0.7	6	5	300	6	0.044	0.121	0.988	0.986
MR	0	RM	0.7	6	15	100	3	0.005	0.572	1.346	0.971
MR	0	RM	0.7	6	15	100	6	0.059	0.569	1.029	1.007
MR	0	RM	0.7	6	15	300	3	0.009	0.345	1.014	1.005
MR	0	RM	0.7	6	15	300	6	0.074	0.369	0.993	1.000
MR	0	RM	0.7	12	5	100	3	0.001	0.210	0.946	0.990
MR	0	RM	0.7	12	5	100	6	0.053	0.222	0.991	0.950
MR	0	RM	0.7	12	5	300	3	0.016	0.190	1.019	0.990
MR	0	RM	0.7	12	5	300	6	0.031	0.162	1.025	0.979
MR	0	RM	0.7	12	15	100	3	0.015	0.882	1.306	1.014
MR	0	RM	0.7	12	15	100	6	0.004	0.634	1.007	0.974
MR	0	RM	0.7	12	15	300	3	0.014	0.563	1.015	0.995
MR	0	RM	0.7	12	15	300	6	0.013	0.487	1.014	0.988

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	2	LC	0.2	6	5	100	3	0.021	0.410	2.371	0.976
MR	2	LC	0.2	6	5	100	6	0.007	0.275	2.035	0.940
MR	2	LC	0.2	6	5	300	3	0.023	0.412	2.386	0.990
MR	2	LC	0.2	6	5	300	6	0.016	0.284	1.997	0.989
MR	2	LC	0.2	6	15	100	3	0.113	0.787	2.002	1.031
MR	2	LC	0.2	6	15	100	6	0.030	0.578	1.844	0.982
MR	2	LC	0.2	6	15	300	3	0.108	0.781	1.991	1.036
MR	2	LC	0.2	6	15	300	6	0.061	0.593	1.795	1.021
MR	2	LC	0.2	12	5	100	3	0.042	0.746	2.433	0.957
MR	2	LC	0.2	12	5	100	6	0.035	0.478	2.016	0.955
MR	2	LC	0.2	12	5	300	3	0.039	0.746	2.347	1.020
MR	2	LC	0.2	12	5	300	6	0.012	0.509	2.063	1.018
MR	2	LC	0.2	12	15	100	3	0.223	1.466	2.084	1.013
MR	2	LC	0.2	12	15	100	6	0.123	0.982	1.829	1.008
MR	2	LC	0.2	12	15	300	3	0.210	1.490	1.985	1.069
MR	2	LC	0.2	12	15	300	6	0.076	0.996	1.819	1.060
MR	2	LC	0.7	6	5	100	3	0.013	0.380	2.246	0.972
MR	2	LC	0.7	6	5	100	6	0.105	0.379	1.906	0.987
MR	2	LC	0.7	6	5	300	3	0.029	0.443	38.38	1.020
MR	2	LC	0.7	6	5	300	6	0.026	0.351	1.885	0.978

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	2	LC	0.7	6	15	100	3	0.060	0.728	1.910	1.010
MR	2	LC	0.7	6	15	100	6	0.116	0.791	1.712	1.042
MR	2	LC	0.7	6	15	300	3	0.083	0.775	12.13	1.056
MR	2	LC	0.7	6	15	300	6	0.050	0.741	1.670	1.012
MR	2	LC	0.7	12	5	100	3	0.029	0.700	2.299	0.933
MR	2	LC	0.7	12	5	100	6	0.454	0.602	1.933	0.980
MR	2	LC	0.7	12	5	300	3	0.037	0.813	2.371	1.014
MR	2	LC	0.7	12	5	300	6	0.161	0.599	1.949	0.979
MR	2	LC	0.7	12	15	100	3	0.164	1.313	1.905	0.981
MR	2	LC	0.7	12	15	100	6	0.899	1.207	1.712	1.037
MR	2	LC	0.7	12	15	300	3	0.163	1.428	1.913	1.055
MR	2	LC	0.7	12	15	300	6	0.230	1.182	1.702	1.023
MR	2	RC	0.2	6	5	100	3	0.035	0.476	8.535	1.019
MR	2	RC	0.2	6	5	100	6	0.012	0.404	8.729	1.012
MR	2	RC	0.2	6	5	300	3	0.020	0.481	8.919	0.998
MR	2	RC	0.2	6	5	300	6	0.010	0.432	9.625	1.018
MR	2	RC	0.2	6	15	100	3	0.058	0.831	4.982	1.090
MR	2	RC	0.2	6	15	100	6	0.036	0.733	5.162	1.099
MR	2	RC	0.2	6	15	300	3	0.084	0.831	5.057	1.035
MR	2	RC	0.2	6	15	300	6	0.024	0.770	5.350	1.047

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	2	RC	0.2	12	5	100	3	0.034	0.751	8.162	0.980
MR	2	RC	0.2	12	5	100	6	0.003	0.570	8.845	1.011
MR	2	RC	0.2	12	5	300	3	0.012	0.795	9.190	1.024
MR	2	RC	0.2	12	5	300	6	0.009	0.613	9.521	1.030
MR	2	RC	0.2	12	15	100	3	0.233	1.399	4.887	1.061
MR	2	RC	0.2	12	15	100	6	0.032	1.093	5.192	1.094
MR	2	RC	0.2	12	15	300	3	0.144	1.513	5.104	1.075
MR	2	RC	0.2	12	15	300	6	0.029	1.130	5.310	1.065
MR	2	RC	0.7	6	5	100	3	0.014	0.528	8.142	0.996
MR	2	RC	0.7	6	5	100	6	0.033	0.772	8.818	1.048
MR	2	RC	0.7	6	5	300	3	0.013	0.532	8.735	1.003
MR	2	RC	0.7	6	5	300	6	0.028	0.802	9.419	0.996
MR	2	RC	0.7	6	15	100	3	0.044	0.816	4.723	1.055
MR	2	RC	0.7	6	15	100	6	0.073	1.267	4.923	1.081
MR	2	RC	0.7	6	15	300	3	0.043	0.787	4.852	1.026
MR	2	RC	0.7	6	15	300	6	0.073	1.293	5.035	1.023
MR	2	RC	0.7	12	5	100	3	0.042	0.780	8.464	0.956
MR	2	RC	0.7	12	5	100	6	0.433	1.018	8.938	1.029
MR	2	RC	0.7	12	5	300	3	0.027	0.866	8.922	1.019
MR	2	RC	0.7	12	5	300	6	0.017	1.021	9.349	0.990

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	2	RC	0.7	12	15	100	3	0.107	1.275	4.805	1.016
MR	2	RC	0.7	12	15	100	6	0.601	1.730	4.947	1.120
MR	2	RC	0.7	12	15	300	3	0.102	1.328	4.912	1.047
MR	2	RC	0.7	12	15	300	6	0.093	1.662	4.998	1.032
MR	2	RM	0.2	6	5	100	3	0.001	0.090	1.032	0.967
MR	2	RM	0.2	6	5	100	6	0.004	0.090	1.037	0.932
MR	2	RM	0.2	6	5	300	3	0.002	0.103	0.957	0.979
MR	2	RM	0.2	6	5	300	6	0.005	0.084	1.038	0.984
MR	2	RM	0.2	6	15	100	3	0.012	0.268	0.964	0.981
MR	2	RM	0.2	6	15	100	6	0.005	0.238	1.070	0.967
MR	2	RM	0.2	6	15	300	3	0.014	0.297	0.981	0.987
MR	2	RM	0.2	6	15	300	6	0.011	0.235	0.996	0.994
MR	2	RM	0.2	12	5	100	3	0.002	0.167	0.896	0.944
MR	2	RM	0.2	12	5	100	6	0.008	0.134	0.964	0.945
MR	2	RM	0.2	12	5	300	3	0.002	0.183	1.042	1.010
MR	2	RM	0.2	12	5	300	6	0.001	0.127	0.982	1.008
MR	2	RM	0.2	12	15	100	3	0.014	0.498	1.000	0.956
MR	2	RM	0.2	12	15	100	6	0.011	0.373	1.009	0.968
MR	2	RM	0.2	12	15	300	3	0.014	0.527	0.995	1.015
MR	2	RM	0.2	12	15	300	6	0.007	0.374	0.982	1.016

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
MR	2	RM	0.7	6	5	100	3	0.001	0.106	1.032	0.963
MR	2	RM	0.7	6	5	100	6	0.047	0.121	1.020	0.980
MR	2	RM	0.7	6	5	300	3	0.034	0.111	0.986	1.012
MR	2	RM	0.7	6	5	300	6	0.002	0.122	1.032	0.975
MR	2	RM	0.7	6	15	100	3	0.005	0.331	1.047	0.981
MR	2	RM	0.7	6	15	100	6	0.029	0.350	1.018	1.016
MR	2	RM	0.7	6	15	300	3	0.026	0.340	1.030	1.020
MR	2	RM	0.7	6	15	300	6	0.005	0.341	0.999	0.991
MR	2	RM	0.7	12	5	100	3	0.019	0.176	0.999	0.922
MR	2	RM	0.7	12	5	100	6	0.082	0.166	0.989	0.970
MR	2	RM	0.7	12	5	300	3	0.006	0.210	1.042	1.003
MR	2	RM	0.7	12	5	300	6	0.030	0.179	1.013	0.971
MR	2	RM	0.7	12	15	100	3	0.007	0.554	1.002	0.936
MR	2	RM	0.7	12	15	100	6	0.474	0.498	1.020	1.001
MR	2	RM	0.7	12	15	300	3	0.012	0.550	1.000	1.007
MR	2	RM	0.7	12	15	300	6	0.085	0.504	1.030	0.982
PP	0	LC	0.2	6	5	100	3	0.016	0.464	0.000	1.012
PP	0	LC	0.2	6	5	100	6	0.005	0.372	0.000	0.984
PP	0	LC	0.2	6	5	300	3	0.023	0.346	0.000	0.988
PP	0	LC	0.2	6	5	300	6	0.011	0.276	0.000	1.023

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	0	LC	0.2	6	15	100	3	0.073	0.810	0.000	1.090
PP	0	LC	0.2	6	15	100	6	0.015	0.672	0.000	1.058
PP	0	LC	0.2	6	15	300	3	0.076	0.612	0.000	1.020
PP	0	LC	0.2	6	15	300	6	0.022	0.518	0.000	1.054
PP	0	LC	0.2	12	5	100	3	0.024	0.667	0.000	1.019
PP	0	LC	0.2	12	5	100	6	0.006	0.518	0.000	0.977
PP	0	LC	0.2	12	5	300	3	0.037	0.657	0.000	1.010
PP	0	LC	0.2	12	5	300	6	0.005	0.450	0.000	1.002
PP	0	LC	0.2	12	15	100	3	0.151	1.371	0.000	1.096
PP	0	LC	0.2	12	15	100	6	0.014	0.934	0.000	1.044
PP	0	LC	0.2	12	15	300	3	0.151	1.160	0.000	1.047
PP	0	LC	0.2	12	15	300	6	0.024	0.828	0.000	1.031
PP	0	LC	0.7	6	5	100	3	0.018	0.510	0.000	0.972
PP	0	LC	0.7	6	5	100	6	0.214	0.590	0.000	0.987
PP	0	LC	0.7	6	5	300	3	0.011	0.370	0.000	1.005
PP	0	LC	0.7	6	5	300	6	0.021	0.434	0.000	0.992
PP	0	LC	0.7	6	15	100	3	0.051	0.714	0.000	1.034
PP	0	LC	0.7	6	15	100	6	0.256	0.913	0.000	1.042
PP	0	LC	0.7	6	15	300	3	0.050	0.591	0.000	1.030
PP	0	LC	0.7	6	15	300	6	0.047	0.823	0.000	1.020

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	0	LC	0.7	12	5	100	3	0.016	0.719	0.000	1.010
PP	0	LC	0.7	12	5	100	6	0.023	0.650	0.000	0.970
PP	0	LC	0.7	12	5	300	3	0.022	0.656	0.000	0.999
PP	0	LC	0.7	12	5	300	6	0.103	0.559	0.000	0.987
PP	0	LC	0.7	12	15	100	3	0.114	1.220	0.000	1.078
PP	0	LC	0.7	12	15	100	6	0.035	1.129	0.000	1.014
PP	0	LC	0.7	12	15	300	3	0.095	1.014	0.000	1.032
PP	0	LC	0.7	12	15	300	6	0.091	1.080	0.000	1.016
PP	0	RC	0.2	6	5	100	3	0.014	0.422	0.000	1.011
PP	0	RC	0.2	6	5	100	6	0.006	0.393	0.000	0.985
PP	0	RC	0.2	6	5	300	3	0.019	0.353	0.000	0.989
PP	0	RC	0.2	6	5	300	6	0.011	0.288	0.000	1.026
PP	0	RC	0.2	6	15	100	3	0.086	0.818	0.000	1.100
PP	0	RC	0.2	6	15	100	6	0.018	0.712	0.000	1.067
PP	0	RC	0.2	6	15	300	3	0.072	0.618	0.000	1.020
PP	0	RC	0.2	6	15	300	6	0.025	0.540	0.000	1.054
PP	0	RC	0.2	12	5	100	3	0.015	0.628	0.000	1.013
PP	0	RC	0.2	12	5	100	6	0.009	0.515	0.000	0.978
PP	0	RC	0.2	12	5	300	3	0.020	0.634	0.000	1.007
PP	0	RC	0.2	12	5	300	6	0.015	0.465	0.000	1.004

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	0	RC	0.2	12	15	100	3	0.148	1.339	0.000	1.090
PP	0	RC	0.2	12	15	100	6	0.036	0.948	0.000	1.033
PP	0	RC	0.2	12	15	300	3	0.116	1.153	0.000	1.044
PP	0	RC	0.2	12	15	300	6	0.060	0.859	0.000	1.040
PP	0	RC	0.7	6	5	100	3	0.005	0.449	0.000	0.967
PP	0	RC	0.7	6	5	100	6	0.213	0.548	0.000	0.988
PP	0	RC	0.7	6	5	300	3	0.012	0.378	0.000	1.008
PP	0	RC	0.7	6	5	300	6	0.083	0.422	0.000	0.993
PP	0	RC	0.7	6	15	100	3	0.019	0.727	0.000	1.009
PP	0	RC	0.7	6	15	100	6	0.067	0.895	0.000	1.043
PP	0	RC	0.7	6	15	300	3	0.048	0.578	0.000	1.034
PP	0	RC	0.7	6	15	300	6	0.095	0.809	0.000	1.020
PP	0	RC	0.7	12	5	100	3	0.016	0.718	0.000	1.004
PP	0	RC	0.7	12	5	100	6	0.010	0.661	0.000	0.972
PP	0	RC	0.7	12	5	300	3	0.052	0.681	0.000	1.002
PP	0	RC	0.7	12	5	300	6	0.043	0.574	0.000	0.987
PP	0	RC	0.7	12	15	100	3	0.133	1.134	0.000	1.067
PP	0	RC	0.7	12	15	100	6	0.018	1.082	0.000	1.024
PP	0	RC	0.7	12	15	300	3	0.158	1.070	0.000	1.032
PP	0	RC	0.7	12	15	300	6	0.090	1.084	0.000	1.016

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	0	RM	0.2	6	5	100	3	0.000	0.085	0.000	0.991
PP	0	RM	0.2	6	5	100	6	0.001	0.065	0.000	0.949
PP	0	RM	0.2	6	5	300	3	0.002	0.037	0.000	0.978
PP	0	RM	0.2	6	5	300	6	0.002	0.032	0.000	1.013
PP	0	RM	0.2	6	15	100	3	0.002	0.153	0.000	1.014
PP	0	RM	0.2	6	15	100	6	0.001	0.119	0.000	0.990
PP	0	RM	0.2	6	15	300	3	0.004	0.068	0.000	0.982
PP	0	RM	0.2	6	15	300	6	0.004	0.056	0.000	1.024
PP	0	RM	0.2	12	5	100	3	0.000	0.095	0.000	1.000
PP	0	RM	0.2	12	5	100	6	0.001	0.085	0.000	0.950
PP	0	RM	0.2	12	5	300	3	0.001	0.059	0.000	0.997
PP	0	RM	0.2	12	5	300	6	0.003	0.048	0.000	0.991
PP	0	RM	0.2	12	15	100	3	0.002	0.268	0.000	1.019
PP	0	RM	0.2	12	15	100	6	0.004	0.168	0.000	0.982
PP	0	RM	0.2	12	15	300	3	0.000	0.109	0.000	1.001
PP	0	RM	0.2	12	15	300	6	0.005	0.086	0.000	1.001
PP	0	RM	0.7	6	5	100	3	0.001	0.087	0.000	0.954
PP	0	RM	0.7	6	5	100	6	0.121	0.330	0.000	1.017
PP	0	RM	0.7	6	5	300	3	0.001	0.042	0.000	0.999
PP	0	RM	0.7	6	5	300	6	0.068	0.036	0.000	0.990

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	0	RM	0.7	6	15	100	3	0.769	0.176	0.000	1.401
PP	0	RM	0.7	6	15	100	6	0.187	0.538	0.000	1.123
PP	0	RM	0.7	6	15	300	3	0.002	0.083	0.000	1.011
PP	0	RM	0.7	6	15	300	6	0.088	0.144	0.000	1.016
PP	0	RM	0.7	12	5	100	3	0.001	0.133	0.000	0.991
PP	0	RM	0.7	12	5	100	6	0.598	0.110	0.000	1.022
PP	0	RM	0.7	12	5	300	3	0.017	0.054	0.000	0.990
PP	0	RM	0.7	12	5	300	6	0.142	0.621	0.000	0.987
PP	0	RM	0.7	12	15	100	3	0.039	0.314	0.000	1.064
PP	0	RM	0.7	12	15	100	6	0.267	0.287	0.000	1.219
PP	0	RM	0.7	12	15	300	3	0.006	0.099	0.000	0.998
PP	0	RM	0.7	12	15	300	6	0.308	1.021	0.000	1.001
PP	2	LC	0.2	6	5	100	3	0.015	0.333	0.000	0.975
PP	2	LC	0.2	6	5	100	6	0.005	0.214	0.000	0.939
PP	2	LC	0.2	6	5	300	3	0.017	0.333	0.000	0.989
PP	2	LC	0.2	6	5	300	6	0.013	0.222	0.000	0.987
PP	2	LC	0.2	6	15	100	3	0.080	0.603	0.000	1.023
PP	2	LC	0.2	6	15	100	6	0.017	0.425	0.000	0.978
PP	2	LC	0.2	6	15	300	3	0.075	0.595	0.000	1.026
PP	2	LC	0.2	6	15	300	6	0.041	0.438	0.000	1.010

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	2	LC	0.2	12	5	100	3	0.035	0.611	0.000	0.957
PP	2	LC	0.2	12	5	100	6	0.029	0.382	0.000	0.955
PP	2	LC	0.2	12	5	300	3	0.032	0.610	0.000	1.019
PP	2	LC	0.2	12	5	300	6	0.010	0.408	0.000	1.017
PP	2	LC	0.2	12	15	100	3	0.181	1.150	0.000	1.009
PP	2	LC	0.2	12	15	100	6	0.092	0.751	0.000	1.008
PP	2	LC	0.2	12	15	300	3	0.168	1.154	0.000	1.062
PP	2	LC	0.2	12	15	300	6	0.055	0.759	0.000	1.051
PP	2	LC	0.7	6	5	100	3	0.011	0.337	0.000	0.970
PP	2	LC	0.7	6	5	100	6	0.169	0.277	0.000	1.002
PP	2	LC	0.7	6	5	300	3	0.027	0.381	0.000	1.021
PP	2	LC	0.7	6	5	300	6	0.024	0.251	0.000	0.976
PP	2	LC	0.7	6	15	100	3	0.041	0.502	0.000	1.006
PP	2	LC	0.7	6	15	100	6	0.264	0.511	0.000	1.034
PP	2	LC	0.7	6	15	300	3	0.062	0.563	0.000	1.052
PP	2	LC	0.7	6	15	300	6	0.033	0.485	0.000	1.007
PP	2	LC	0.7	12	5	100	3	0.025	0.559	0.000	0.932
PP	2	LC	0.7	12	5	100	6	0.105	0.453	0.000	0.980
PP	2	LC	0.7	12	5	300	3	0.032	0.660	0.000	1.013
PP	2	LC	0.7	12	5	300	6	0.134	0.456	0.000	0.977

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	2	LC	0.7	12	15	100	3	0.132	0.944	0.000	0.978
PP	2	LC	0.7	12	15	100	6	0.129	0.850	0.000	1.118
PP	2	LC	0.7	12	15	300	3	0.135	1.067	0.000	1.049
PP	2	LC	0.7	12	15	300	6	0.266	0.847	0.000	1.015
PP	2	RC	0.2	6	5	100	3	0.030	0.407	0.000	1.021
PP	2	RC	0.2	6	5	100	6	0.012	0.353	0.000	1.019
PP	2	RC	0.2	6	5	300	3	0.014	0.409	0.000	0.987
PP	2	RC	0.2	6	5	300	6	0.009	0.381	0.000	1.019
PP	2	RC	0.2	6	15	100	3	0.037	0.666	0.000	1.110
PP	2	RC	0.2	6	15	100	6	0.032	0.616	0.000	1.141
PP	2	RC	0.2	6	15	300	3	0.056	0.663	0.000	1.031
PP	2	RC	0.2	6	15	300	6	0.017	0.654	0.000	1.055
PP	2	RC	0.2	12	5	100	3	0.028	0.619	0.000	0.982
PP	2	RC	0.2	12	5	100	6	0.002	0.482	0.000	1.019
PP	2	RC	0.2	12	5	300	3	0.008	0.665	0.000	1.024
PP	2	RC	0.2	12	5	300	6	0.008	0.520	0.000	1.031
PP	2	RC	0.2	12	15	100	3	0.206	1.084	0.000	1.074
PP	2	RC	0.2	12	15	100	6	0.025	0.887	0.000	1.131
PP	2	RC	0.2	12	15	300	3	0.108	1.189	0.000	1.076
PP	2	RC	0.2	12	15	300	6	0.015	0.921	0.000	1.073

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	2	RC	0.7	6	5	100	3	0.013	0.456	0.000	0.996
PP	2	RC	0.7	6	5	100	6	0.034	0.698	0.000	1.052
PP	2	RC	0.7	6	5	300	3	0.010	0.459	0.000	1.002
PP	2	RC	0.7	6	5	300	6	0.030	0.731	0.000	2.989
PP	2	RC	0.7	6	15	100	3	0.043	0.646	0.000	1.071
PP	2	RC	0.7	6	15	100	6	0.082	1.113	0.000	1.105
PP	2	RC	0.7	6	15	300	3	0.034	0.607	0.000	1.026
PP	2	RC	0.7	6	15	300	6	0.076	1.148	0.000	3.078
PP	2	RC	0.7	12	5	100	3	0.038	0.656	0.000	0.958
PP	2	RC	0.7	12	5	100	6	0.419	0.901	0.000	1.034
PP	2	RC	0.7	12	5	300	3	0.023	0.726	0.000	1.018
PP	2	RC	0.7	12	5	300	6	0.014	0.908	0.000	0.990
PP	2	RC	0.7	12	15	100	3	0.087	0.943	0.000	1.031
PP	2	RC	0.7	12	15	100	6	0.700	1.485	0.000	1.150
PP	2	RC	0.7	12	15	300	3	0.084	0.990	0.000	1.048
PP	2	RC	0.7	12	15	300	6	0.094	1.426	0.000	1.032
PP	2	RM	0.2	6	5	100	3	0.001	0.060	0.000	0.968
PP	2	RM	0.2	6	5	100	6	0.004	0.058	0.000	0.932
PP	2	RM	0.2	6	5	300	3	0.000	0.034	0.000	0.979
PP	2	RM	0.2	6	5	300	6	0.004	0.034	0.000	0.984

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	2	RM	0.2	6	15	100	3	0.003	0.116	0.000	0.983
PP	2	RM	0.2	6	15	100	6	0.003	0.114	0.000	0.986
PP	2	RM	0.2	6	15	300	3	0.001	0.063	0.000	0.984
PP	2	RM	0.2	6	15	300	6	0.005	0.063	0.000	0.996
PP	2	RM	0.2	12	5	100	3	0.001	0.105	0.000	0.944
PP	2	RM	0.2	12	5	100	6	0.008	0.080	0.000	0.947
PP	2	RM	0.2	12	5	300	3	0.001	0.062	0.000	1.010
PP	2	RM	0.2	12	5	300	6	0.000	0.037	0.000	1.008
PP	2	RM	0.2	12	15	100	3	0.005	0.200	0.000	0.959
PP	2	RM	0.2	12	15	100	6	0.005	0.150	0.000	0.981
PP	2	RM	0.2	12	15	300	3	0.002	0.108	0.000	1.015
PP	2	RM	0.2	12	15	300	6	0.001	0.086	0.000	1.019
PP	2	RM	0.7	6	5	100	3	0.001	0.066	0.000	0.965
PP	2	RM	0.7	6	5	100	6	1.719	0.074	0.000	1.101
PP	2	RM	0.7	6	5	300	3	0.037	0.039	0.000	1.012
PP	2	RM	0.7	6	5	300	6	3.152	0.050	0.000	1.008
PP	2	RM	0.7	6	15	100	3	0.058	0.143	0.000	1.043
PP	2	RM	0.7	6	15	100	6	1.024	0.142	0.000	1.508
PP	2	RM	0.7	6	15	300	3	0.038	0.078	0.000	1.028
PP	2	RM	0.7	6	15	300	6	1.638	0.118	0.000	1.414

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
PP	2	RM	0.7	12	5	100	3	0.042	0.096	0.000	0.924
PP	2	RM	0.7	12	5	100	6	1.812	0.094	0.000	1.036
PP	2	RM	0.7	12	5	300	3	0.009	0.071	0.000	1.004
PP	2	RM	0.7	12	5	300	6	1.489	0.064	0.000	1.019
PP	2	RM	0.7	12	15	100	3	0.782	0.202	0.000	1.342
PP	2	RM	0.7	12	15	100	6	0.540	0.226	0.000	1.365
PP	2	RM	0.7	12	15	300	3	0.014	0.116	0.000	1.012
PP	2	RM	0.7	12	15	300	6	2.698	0.122	0.000	1.025
RG	0	LC	0.2	6	5	100	3	0.003	0.504	3.826	1.001
RG	0	LC	0.2	6	5	100	6	0.004	0.396	3.469	0.964
RG	0	LC	0.2	6	5	300	3	0.006	0.389	4.007	0.982
RG	0	LC	0.2	6	5	300	6	0.006	0.306	3.579	1.019
RG	0	LC	0.2	6	15	100	3	0.013	0.881	3.449	1.042
RG	0	LC	0.2	6	15	100	6	0.028	0.733	3.103	1.033
RG	0	LC	0.2	6	15	300	3	0.007	0.680	2.965	0.993
RG	0	LC	0.2	6	15	300	6	0.016	0.578	3.112	1.050
RG	0	LC	0.2	12	5	100	3	0.006	0.759	3.977	1.013
RG	0	LC	0.2	12	5	100	6	0.005	0.589	3.586	0.972
RG	0	LC	0.2	12	5	300	3	0.010	0.765	4.238	1.005
RG	0	LC	0.2	12	5	300	6	0.002	0.524	3.765	1.000

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	0	LC	0.2	12	15	100	3	0.005	1.604	3.858	1.059
RG	0	LC	0.2	12	15	100	6	0.021	1.080	3.355	1.046
RG	0	LC	0.2	12	15	300	3	0.021	1.352	3.091	1.021
RG	0	LC	0.2	12	15	300	6	0.007	0.963	3.141	1.020
RG	0	LC	0.7	6	5	100	3	0.016	0.514	1.962	0.965
RG	0	LC	0.7	6	5	100	6	0.349	0.663	3.798	1.152
RG	0	LC	0.7	6	5	300	3	0.013	0.400	2.107	1.003
RG	0	LC	0.7	6	5	300	6	0.229	0.462	1.060	1.021
RG	0	LC	0.7	6	15	100	3	0.008	0.712	2.169	0.982
RG	0	LC	0.7	6	15	100	6	3.893	1.113	4.490	1.530
RG	0	LC	0.7	6	15	300	3	0.021	0.653	1.831	1.010
RG	0	LC	0.7	6	15	300	6	0.446	0.881	0.998	1.054
RG	0	LC	0.7	12	5	100	3	0.018	0.800	2.166	1.001
RG	0	LC	0.7	12	5	100	6	0.347	0.743	3.321	1.047
RG	0	LC	0.7	12	5	300	3	0.015	0.759	2.533	0.998
RG	0	LC	0.7	12	5	300	6	0.051	0.624	1.109	1.001
RG	0	LC	0.7	12	15	100	3	0.046	1.456	2.692	1.038
RG	0	LC	0.7	12	15	100	6	1.715	1.333	3.353	1.208
RG	0	LC	0.7	12	15	300	3	0.027	1.236	2.048	1.011
RG	0	LC	0.7	12	15	300	6	0.136	1.221	1.120	1.037

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	0	RC	0.2	6	5	100	3	0.030	0.499	3.309	1.019
RG	0	RC	0.2	6	5	100	6	0.037	0.441	2.820	1.006
RG	0	RC	0.2	6	5	300	3	0.022	0.415	3.474	0.994
RG	0	RC	0.2	6	5	300	6	0.031	0.332	2.818	1.037
RG	0	RC	0.2	6	15	100	3	0.131	1.051	3.021	1.129
RG	0	RC	0.2	6	15	100	6	0.098	0.850	2.515	1.128
RG	0	RC	0.2	6	15	300	3	0.066	0.773	2.542	1.034
RG	0	RC	0.2	6	15	300	6	0.074	0.657	2.387	1.091
RG	0	RC	0.2	12	5	100	3	0.021	0.744	3.475	1.017
RG	0	RC	0.2	12	5	100	6	0.034	0.605	3.189	0.988
RG	0	RC	0.2	12	5	300	3	0.026	0.767	3.739	1.011
RG	0	RC	0.2	12	5	300	6	0.032	0.554	3.123	1.012
RG	0	RC	0.2	12	15	100	3	0.139	1.742	3.204	1.105
RG	0	RC	0.2	12	15	100	6	0.118	1.171	2.852	1.101
RG	0	RC	0.2	12	15	300	3	0.099	1.469	2.756	1.052
RG	0	RC	0.2	12	15	300	6	0.081	1.068	2.621	1.061
RG	0	RC	0.7	6	5	100	3	0.055	0.507	2.559	1.001
RG	0	RC	0.7	6	5	100	6	0.721	0.672	6.513	1.352
RG	0	RC	0.7	6	5	300	3	0.045	0.430	2.845	1.030
RG	0	RC	0.7	6	5	300	6	0.769	0.479	1.578	1.085

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	0	RC	0.7	6	15	100	3	0.249	0.917	2.670	1.141
RG	0	RC	0.7	6	15	100	6	3.473	1.317	6.339	1.978
RG	0	RC	0.7	6	15	300	3	0.129	0.727	2.208	1.095
RG	0	RC	0.7	6	15	300	6	1.556	0.976	1.141	1.304
RG	0	RC	0.7	12	5	100	3	0.071	0.832	2.863	1.026
RG	0	RC	0.7	12	5	100	6	1.144	0.774	4.704	1.131
RG	0	RC	0.7	12	5	300	3	0.089	0.809	2.697	1.010
RG	0	RC	0.7	12	5	300	6	0.076	0.661	1.335	1.030
RG	0	RC	0.7	12	15	100	3	0.387	1.613	2.862	1.145
RG	0	RC	0.7	12	15	100	6	1.216	1.457	5.003	1.463
RG	0	RC	0.7	12	15	300	3	0.246	1.391	2.077	1.053
RG	0	RC	0.7	12	15	300	6	0.411	1.328	1.122	1.165
RG	0	RM	0.2	6	5	100	3	0.002	0.145	1.833	0.994
RG	0	RM	0.2	6	5	100	6	0.011	0.136	2.518	0.965
RG	0	RM	0.2	6	5	300	3	0.003	0.087	1.545	0.980
RG	0	RM	0.2	6	5	300	6	0.008	0.084	2.046	1.018
RG	0	RM	0.2	6	15	100	3	0.043	0.451	2.372	1.046
RG	0	RM	0.2	6	15	100	6	0.091	0.408	3.509	1.108
RG	0	RM	0.2	6	15	300	3	0.019	0.247	1.586	0.997
RG	0	RM	0.2	6	15	300	6	0.050	0.239	2.228	1.069

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	0	RM	0.2	12	5	100	3	0.003	0.175	1.578	1.002
RG	0	RM	0.2	12	5	100	6	0.006	0.177	2.238	0.961
RG	0	RM	0.2	12	5	300	3	0.003	0.173	1.493	1.000
RG	0	RM	0.2	12	5	300	6	0.011	0.130	1.804	0.994
RG	0	RM	0.2	12	15	100	3	0.035	0.779	2.442	1.056
RG	0	RM	0.2	12	15	100	6	0.055	0.491	2.750	1.045
RG	0	RM	0.2	12	15	300	3	0.034	0.501	1.643	1.016
RG	0	RM	0.2	12	15	300	6	0.029	0.362	2.023	1.026
RG	0	RM	0.7	6	5	100	3	0.012	0.125	0.806	0.955
RG	0	RM	0.7	6	5	100	6	0.050	0.153	0.557	0.985
RG	0	RM	0.7	6	5	300	3	0.007	0.094	0.780	1.000
RG	0	RM	0.7	6	5	300	6	0.816	0.093	0.318	1.003
RG	0	RM	0.7	6	15	100	3	0.165	0.462	1.433	1.040
RG	0	RM	0.7	6	15	100	6	2.930	0.471	1.310	1.416
RG	0	RM	0.7	6	15	300	3	0.080	0.283	0.876	1.028
RG	0	RM	0.7	6	15	300	6	1.521	0.293	0.591	1.233
RG	0	RM	0.7	12	5	100	3	0.005	0.191	0.825	0.992
RG	0	RM	0.7	12	5	100	6	0.249	0.177	0.364	0.974
RG	0	RM	0.7	12	5	300	3	0.015	0.171	0.733	0.992
RG	0	RM	0.7	12	5	300	6	0.041	0.139	0.296	0.988

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	0	RM	0.7	12	15	100	3	0.142	0.792	1.470	1.074
RG	0	RM	0.7	12	15	100	6	0.125	0.541	0.846	1.210
RG	0	RM	0.7	12	15	300	3	0.039	0.518	0.867	1.019
RG	0	RM	0.7	12	15	300	6	0.423	0.427	0.516	1.170
RG	2	LC	0.2	6	5	100	3	0.005	0.379	2.544	0.968
RG	2	LC	0.2	6	5	100	6	0.004	0.250	2.397	0.937
RG	2	LC	0.2	6	5	300	3	0.004	0.383	2.443	0.982
RG	2	LC	0.2	6	5	300	6	0.012	0.261	2.077	0.985
RG	2	LC	0.2	6	15	100	3	0.010	0.674	2.247	0.982
RG	2	LC	0.2	6	15	100	6	0.006	0.494	2.678	0.966
RG	2	LC	0.2	6	15	300	3	0.003	0.672	2.154	0.986
RG	2	LC	0.2	6	15	300	6	0.026	0.512	2.336	0.994
RG	2	LC	0.2	12	5	100	3	0.012	0.713	2.814	0.952
RG	2	LC	0.2	12	5	100	6	0.016	0.457	2.344	0.951
RG	2	LC	0.2	12	5	300	3	0.012	0.719	2.543	1.015
RG	2	LC	0.2	12	5	300	6	0.006	0.486	2.059	1.014
RG	2	LC	0.2	12	15	100	3	0.028	1.323	2.508	0.969
RG	2	LC	0.2	12	15	100	6	0.012	0.892	2.619	0.982
RG	2	LC	0.2	12	15	300	3	0.030	1.356	2.257	1.025
RG	2	LC	0.2	12	15	300	6	0.018	0.902	2.195	1.022

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	2	LC	0.7	6	5	100	3	0.012	0.344	1.665	0.966
RG	2	LC	0.7	6	5	100	6	0.117	0.340	0.802	0.997
RG	2	LC	0.7	6	5	300	3	0.021	0.423	39.80	1.018
RG	2	LC	0.7	6	5	300	6	0.215	0.315	0.776	0.959
RG	2	LC	0.7	6	15	100	3	0.020	0.607	2.036	0.981
RG	2	LC	0.7	6	15	100	6	0.211	0.668	0.796	1.044
RG	2	LC	0.7	6	15	300	3	0.032	0.673	13.48	1.026
RG	2	LC	0.7	6	15	300	6	0.074	0.625	0.762	1.020
RG	2	LC	0.7	12	5	100	3	0.024	0.669	2.460	0.927
RG	2	LC	0.7	12	5	100	6	0.037	0.563	0.975	0.975
RG	2	LC	0.7	12	5	300	3	0.025	0.781	2.187	1.010
RG	2	LC	0.7	12	5	300	6	0.034	0.564	0.874	0.983
RG	2	LC	0.7	12	15	100	3	0.075	1.193	2.646	0.953
RG	2	LC	0.7	12	15	100	6	0.550	1.067	0.961	1.020
RG	2	LC	0.7	12	15	300	3	0.063	1.303	2.236	1.026
RG	2	LC	0.7	12	15	300	6	0.071	1.053	0.878	1.010
RG	2	RC	0.2	6	5	100	3	0.031	0.455	6.529	1.024
RG	2	RC	0.2	6	5	100	6	0.031	0.382	5.886	1.031
RG	2	RC	0.2	6	5	300	3	0.013	0.462	6.566	1.001
RG	2	RC	0.2	6	5	300	6	0.012	0.413	6.202	1.017

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	2	RC	0.2	6	15	100	3	0.051	0.807	3.964	1.147
RG	2	RC	0.2	6	15	100	6	0.165	0.718	4.030	1.303
RG	2	RC	0.2	6	15	300	3	0.036	0.809	3.787	1.045
RG	2	RC	0.2	6	15	300	6	0.040	0.755	3.947	1.112
RG	2	RC	0.2	12	5	100	3	0.034	0.737	6.474	0.996
RG	2	RC	0.2	12	5	100	6	0.019	0.553	6.438	1.029
RG	2	RC	0.2	12	5	300	3	0.019	0.781	7.066	1.030
RG	2	RC	0.2	12	5	300	6	0.011	0.600	6.809	1.039
RG	2	RC	0.2	12	15	100	3	0.200	1.385	4.028	1.134
RG	2	RC	0.2	12	15	100	6	0.139	1.077	4.273	1.220
RG	2	RC	0.2	12	15	300	3	0.086	1.501	3.995	1.117
RG	2	RC	0.2	12	15	300	6	0.046	1.117	4.133	1.154
RG	2	RC	0.7	6	5	100	3	0.063	0.453	3.848	1.020
RG	2	RC	0.7	6	5	100	6	0.213	0.720	3.048	1.248
RG	2	RC	0.7	6	5	300	3	0.043	0.447	3.705	1.016
RG	2	RC	0.7	6	5	300	6	0.176	0.759	3.319	1.149
RG	2	RC	0.7	6	15	100	3	0.185	0.719	2.563	1.177
RG	2	RC	0.7	6	15	100	6	0.890	1.220	2.075	1.760
RG	2	RC	0.7	6	15	300	3	0.069	0.671	2.245	1.071
RG	2	RC	0.7	6	15	300	6	1.404	1.253	2.061	1.511

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	2	RC	0.7	12	5	100	3	0.089	0.728	4.252	0.972
RG	2	RC	0.7	12	5	100	6	0.248	0.974	3.128	1.138
RG	2	RC	0.7	12	5	300	3	0.040	0.819	4.103	1.027
RG	2	RC	0.7	12	5	300	6	0.191	0.987	3.326	1.090
RG	2	RC	0.7	12	15	100	3	0.204	1.216	3.065	1.116
RG	2	RC	0.7	12	15	100	6	0.924	1.687	2.129	1.438
RG	2	RC	0.7	12	15	300	3	0.116	1.271	2.675	1.115
RG	2	RC	0.7	12	15	300	6	1.297	1.624	2.101	1.311
RG	2	RM	0.2	6	5	100	3	0.004	0.093	1.904	0.972
RG	2	RM	0.2	6	5	100	6	0.009	0.097	2.335	0.943
RG	2	RM	0.2	6	5	300	3	0.002	0.094	1.563	0.980
RG	2	RM	0.2	6	5	300	6	0.009	0.084	2.169	0.990
RG	2	RM	0.2	6	15	100	3	0.022	0.252	1.843	1.003
RG	2	RM	0.2	6	15	100	6	0.038	0.273	3.079	1.080
RG	2	RM	0.2	6	15	300	3	0.018	0.265	1.600	0.997
RG	2	RM	0.2	6	15	300	6	0.093	0.239	2.384	1.042
RG	2	RM	0.2	12	5	100	3	0.008	0.175	1.700	0.948
RG	2	RM	0.2	12	5	100	6	0.008	0.149	2.152	0.955
RG	2	RM	0.2	12	5	300	3	0.007	0.178	1.746	1.012
RG	2	RM	0.2	12	5	300	6	0.003	0.125	1.725	1.012

Table 30--continued.

M	SK	TM	R	E	PM	N	p	BSE	D	Q	RE
RG	2	RM	0.2	12	15	100	3	0.053	0.511	2.255	0.989
RG	2	RM	0.2	12	15	100	6	0.087	0.420	3.042	1.060
RG	2	RM	0.2	12	15	300	3	0.036	0.507	1.693	1.028
RG	2	RM	0.2	12	15	300	6	0.026	0.375	2.038	1.053
RG	2	RM	0.7	6	5	100	3	0.004	0.087	0.768	0.966
RG	2	RM	0.7	6	5	100	6	0.820	0.093	0.416	0.993
RG	2	RM	0.7	6	5	300	3	0.007	0.090	0.739	1.013
RG	2	RM	0.7	6	5	300	6	0.153	0.092	0.350	0.984
RG	2	RM	0.7	6	15	100	3	0.098	0.279	1.068	1.022
RG	2	RM	0.7	6	15	100	6	1.061	0.291	0.896	1.287
RG	2	RM	0.7	6	15	300	3	0.028	0.278	0.823	1.036
RG	2	RM	0.7	6	15	300	6	2.386	0.268	0.613	1.155
RG	2	RM	0.7	12	5	100	3	0.009	0.163	0.891	0.925
RG	2	RM	0.7	12	5	100	6	0.056	0.141	0.465	0.982
RG	2	RM	0.7	12	5	300	3	0.002	0.191	0.750	1.004
RG	2	RM	0.7	12	5	300	6	0.541	0.155	0.343	0.985
RG	2	RM	0.7	12	15	100	3	0.054	0.514	1.015	0.975
RG	2	RM	0.7	12	15	100	6	1.430	0.448	0.912	1.263
RG	2	RM	0.7	12	15	300	3	0.094	0.505	0.860	1.027
RG	2	RM	0.7	12	15	300	6	2.436	0.442	0.545	1.147

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